Vanguard Life-Cycle Investing Model: A framework for building target-date portfolios

Vanguard Research

September 2016

Roger Aliaga-Díaz, Ph.D.; Harshdeep Ahluwalia, M.Sc.; Kevin DiCiurcio, CFA; Matthew C. Brancato, CFA, CPA; Scott Donaldson, CFA, CFP®; David Pakula, CFA

- The Vanguard Life-Cycle Investing Model (VLCM) is a proprietary model for glide-path construction that can assist in creating custom investment portfolios for retirement and is driven by the demographic characteristics and investment constraints that are unique to the average participant population.
- The VLCM embodies key principles of life-cycle investing theory, including a utility framework and plan risk aversion. This enables cost-benefit analysis of glide-path customization, evaluation of risk–return trade-offs of various asset allocation choices, and simulation of portfolio performance under different market scenarios.
- Our analysis finds that homogeneous participant populations that differ significantly from the broad population in their degree of risk aversion, their retirement age, or the presence of a defined benefit (DB) plan are better served by glide-path customization. Simulations based on the VLCM show that taking more equity risk in the glide path does not substitute for adequate savings. In fact, model-derived custom glide paths for populations with various levels of savings do not meaningfully differ.
- We find that other factors such as investment costs, retirement age, and retirement spending (replacement ratio) better improve the odds of retirement income sufficiency than do sub-asset allocation tilts, including the addition of alternatives.

Life-cycle investing is the study of investors' decisions about saving and investing for retirement. Life-cycle models can be used to determine whether investors are saving enough during their working life while investing appropriately to generate required income during retirement.

It is well-known that because of either behavioral biases or the complexity of analysis, investors may choose not to, or be unable to, go through this retirement planning exercise. Target-date funds (TDFs) have addressed many aspects of this challenge by simplifying the asset allocation decision (i.e., setting up a glide path). The glide path defines a TDF's level of risk. Investing in a TDF delegates subsequent decisions about portfolio construction and life-cycle rebalancing to the fund's portfolio manager. As a result, the adoption of TDFs in employer-sponsored and individual retirement plans has increased exponentially over the past ten years.

As TDF strategies have become more prominent in investment menus, interest has grown in customizing TDF glide paths, based on the plan sponsor's distinct investment beliefs, the unique characteristics of the plan participant population, or a preference for more direct control over plan investment strategy. In its own TDF guidelines for sponsors, the U.S. Department of Labor suggested that sponsors ask their investment providers about custom versus proprietary offerings.[1] Depending on the plan fiduciaries' judgment and each plan's individual circumstances, either off-the-shelf or customized options could be acceptable fiduciary choices.

However, the simplicity and practicality of off-the-shelf TDFs could come at the expense of the ability to accommodate a **participant population's average characteristics** and investment constraints. Although retirement goals may be similar across all plans, constraints—such as risk tolerance, required replacement ratios, income during working years, saving rates, presence of out-of-plan assets, and expectations for Social Security income—may each justify some degree of portfolio customization. A more customized glide path may add value whenever the characteristics of a homogeneous participant population significantly differ from those of the broad population.[2] To be clear, this is customization for the average participant population, rather than at an individual level. Importantly, the benefit of customization needs to be weighed against the cost of implementation. A summary of factors affecting the retirement income asset allocation (glide path) is shown in **Table A-1** on page 17 in the Appendix.

A model of optimal life-cycle investing helps assess the cost-benefit analysis of customization, taking into account all these factors. The model is critical in building a consistent and objective retirement investment framework, including estimating retirement income feasibility and savings level sufficiency and quantifying the risk–return trade-offs of various customized glide paths. Although the VLCM is used to customize a glide path for a participant population, it can also be applied at an individual level, at least in theory. For instance, it can be used to customize portfolio options to meet the specific needs of individual participants in a managed account or personalized advice service.

1 See U.S. Department of Labor (2013).

2 Heterogeneous populations—those with various subpopulations—can be a challenge to glide-path customization, because the glide path is designed for an average participant, whereas a heterogeneous population may have several participants who differ significantly from the average one. Hence, a single custom glide path may not be well-suited for all participants in heterogeneous populations.

The shape of the glide path

A glide path is the distinguishing feature of a target-date fund. Practitioners often use equity holdings or allocations to represent the glide path, but a more robust definition includes all risky assets. Although there is no uniform methodology to constructing glide paths, they have several commonalities in practice. In terms of the slope of the equity allocation, all 60-plus TDFs in the industry are downward-sloping. As to the equity level, 80% of the assets in TDFs reside with providers whose glide paths are within a 15 percentage point range of the industry average.[3]

Outside the commercial TDF industry, a wealth of literature about life-cycle investing reflects less agreement on the optimal shape of the glide path. Bodie, Merton, and Samuelson (1992) and Gomes, Kotlikoff, and Viceira (2008) argued for a downward slope, while Shiller (2005), Basu and Drew (2009), Arnott (2012), and Arnott, Sherrerd, and Wu (2013) said a rising glide path was better. Pfau and Kitces (2014) argued for a U-shaped glide path with an inflection point at retirement, and Estrada (2016) argued for an inverted U-shaped glide path with an inflection point at retirement.[4]

The Vanguard Life-Cycle Investing Model

The VLCM is a proprietary model developed by Vanguard's Investment Strategy Group. It has several practical benefits.

1. It provides a rigorous quantitative framework for constructing custom glide paths based on the specific characteristics of the average participant population (as listed in Table A-1).
2. It quantifies the benefits of customization to a participant population based on its average risk tolerance and investment constraints using a utility function-based framework. Any glide-path customization analysis should be done in the context of incremental costs and benefits versus an existing off-the-shelf TDF.[5]
3. It permits calculating a wide range of portfolio metrics that illustrate the main risk–return trade-offs involved in choosing among different glide paths or portfolios.
4. Combined with long-term asset return expectations derived from the Vanguard Capital Markets Model® (VCMM), the VLCM is a powerful simulation tool for retirement portfolios under various market scenarios or participants' changing economic conditions, including the calculation of key metrics of investment success, such as retirement income sufficiency and longevity risk. Investment success can also be defined as a low probability of depleting financial wealth by age 95.
5. The VLCM can facilitate a deeper understanding of an existing TDF in the context of regular due diligence. This process is an important element of the ongoing oversight that a plan sponsor should perform.[5]

3 Vanguard analysis of data from Morningstar Direct, 2016.

4 See also Basu, Doran, and Drew (2012).

5 Brancato et al. (2014).

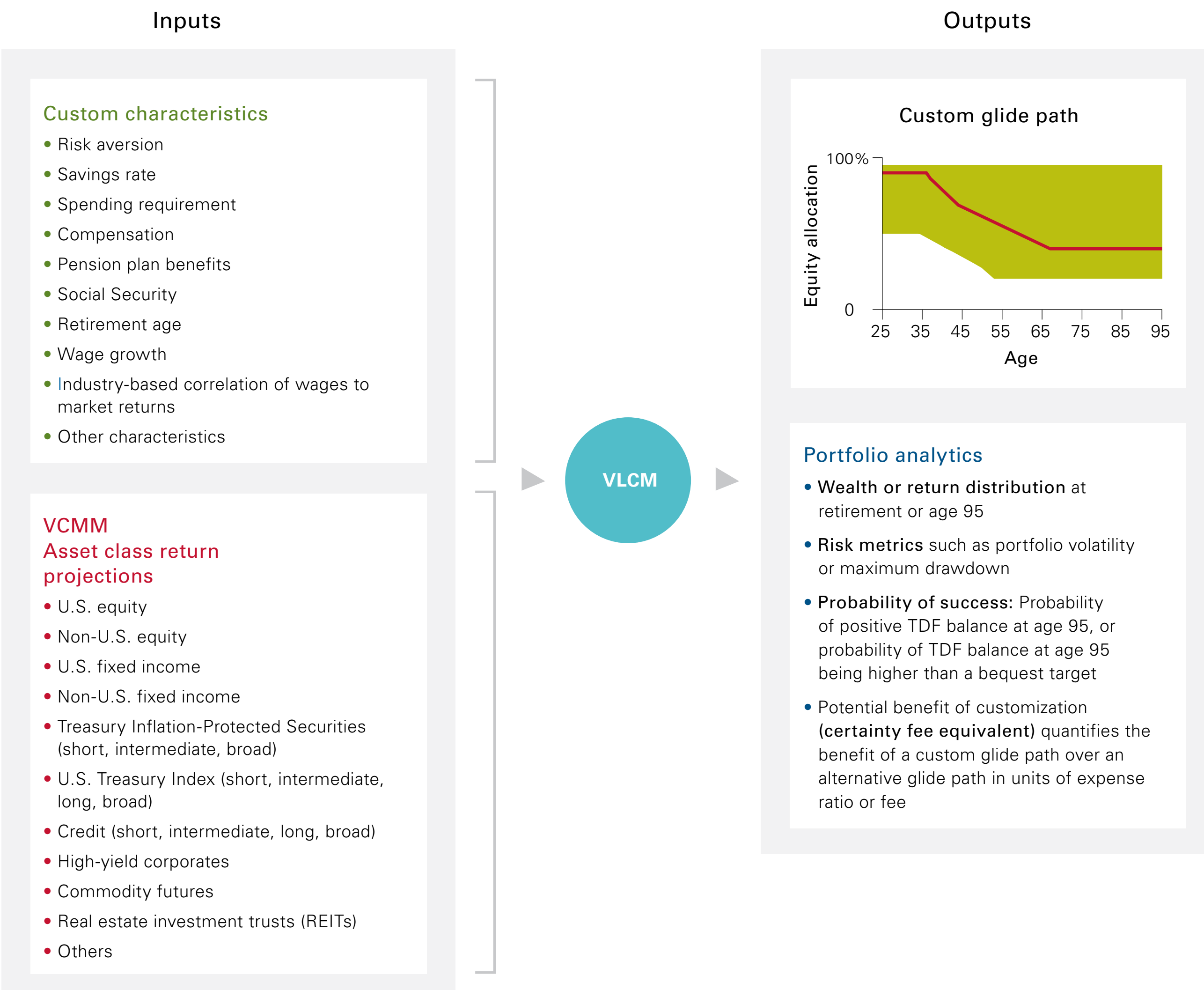
The VLCM framework

The VLCM generates an optimal custom glide path for a participant population by assessing the trade-offs between the expected (mean) wealth accumulation and the uncertainty about that wealth outcome, for thousands of potential glide paths. As shown in **Figure 1**, the VLCM does this by combining two sets of inputs: the **asset class return projections** from our proprietary VCMM and the average **characteristics of the participant population**.

Along with the optimal custom glide path, the VLCM generates a wide range of portfolio metrics such as a distribution of potential wealth accumulation outcomes, risk and return distributions for the asset allocation, and probability of ruin, such as the odds of participants depleting their wealth by age 95.

The VLCM inherits the distributional forecasting framework of the VCMM (see Davis et al., 2014) and applies to it the calculation of wealth outcomes from

Figure 1. The VLCM process

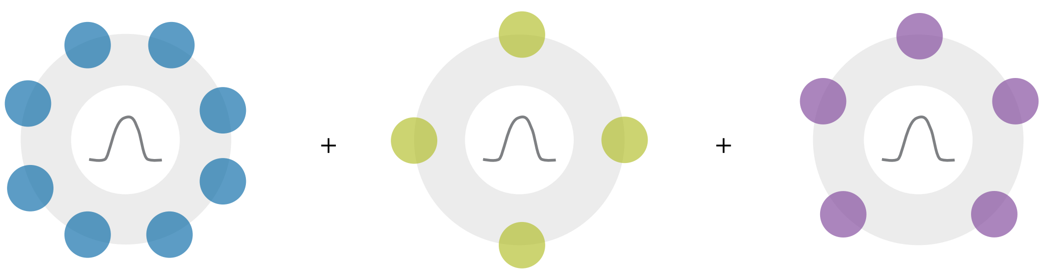


Source: Vanguard.

any given portfolio. The VLCM determines the total wealth distribution post-retirement for each potential glide path. In designing the glide path, it's important to use a broader retirement wealth concept.

As shown in **Figure 2**, the wealth of an average plan participant post-retirement comprises retirement investments (such as a TDF), Social Security, and a pension plan (if applicable). Income from sources such as Social Security and a pension plan could potentially allow a retiree to take on more or less risk with his or her retirement investments and hence is needed in deriving the custom investment plan. Several factors such as contribution (savings) rate, glide path, time horizon (retirement age), and spending requirements also determine his or her wealth distribution in addition to factors beyond one's control, such as market returns.

Figure 2. Determining total wealth distribution



TDF wealth distribution

- Time horizon
- Spending requirement
- Contribution rate
- VCMM—asset return projections
- Glide path
- Compensation growth (serial and cross-sectional)
- Compensation level and scale (participant population-based)
- Correlation of compensation with markets

Social Security wealth distribution

- Retirement age
- Compensation level and scale (participant population-based)
- Mortality rates
- VCMM—interest rate projections

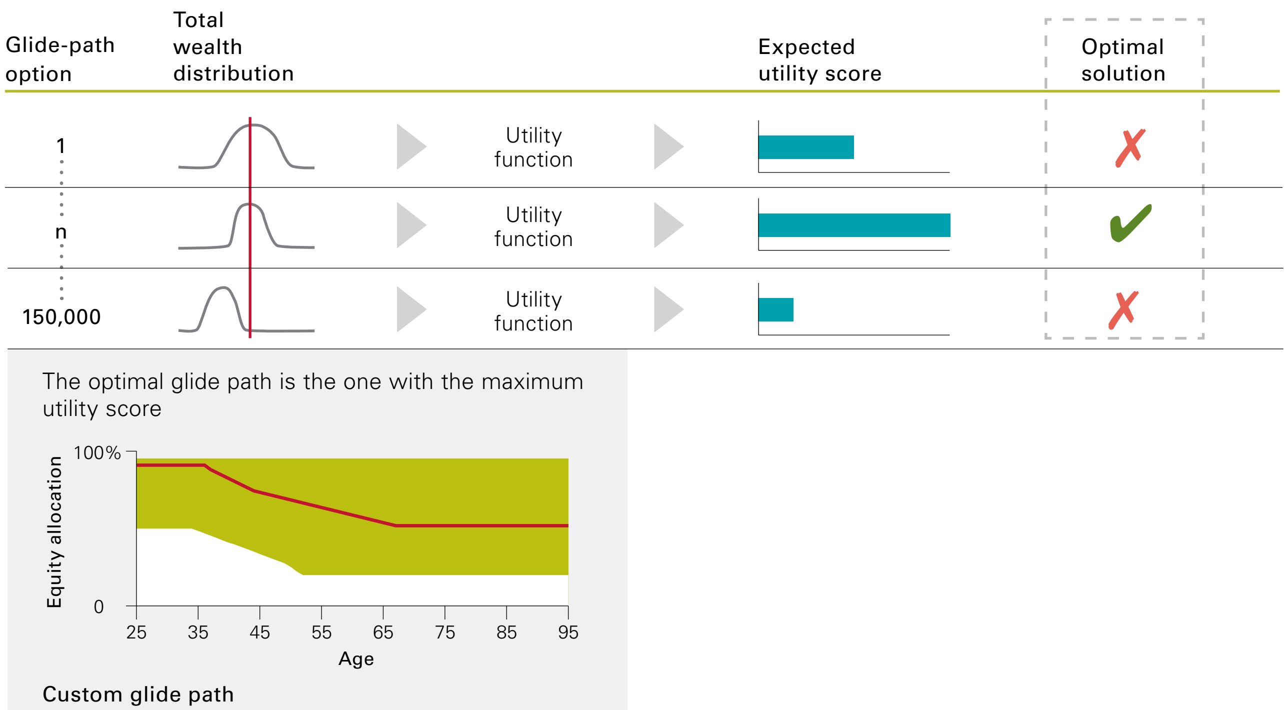
Pension plan wealth distribution

- Retirement age
- Compensation level and scale (participant population-based)
- Pension plan specifics
- VCMM—interest rate projections
- Mortality rates

Source: Vanguard.

Comparing glide paths by their associated wealth distributions post-retirement requires deciding among risk–return trade-offs. Higher expected wealth glide paths normally come with more uncertainty about that average number. The VLCM ranks different glide-path options (and the resulting wealth distribution) by applying the risk-tolerance criteria embedded in the so-called utility function. More simply, the function works as a scoring system that solves the risk–return trade-off inherent in different investment options. Thousands of glide paths result in thousands of utility scores, and the glide path with the highest score—the one that strikes the right balance between the glide path's expected return and risk—is the best option given the population characteristics (see **Figure 3**). A brief technical presentation of the model is included on page 18 in the Appendix.

Figure 3. The VLCM framework



Note: The green shaded area represents the potential range of glide-path options.
Source: Vanguard.

Investing is about the journey, not just the destination

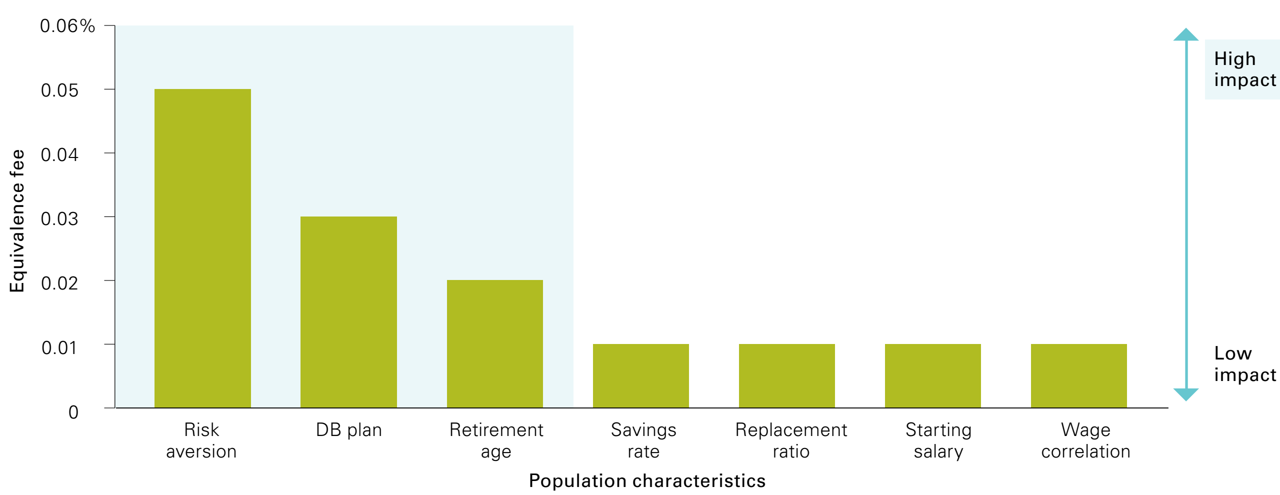
In retirement investing, as in portfolio construction, it is never the goal to achieve maximum returns or maximum wealth at retirement with no consideration for the portfolio's risk profile.[6] The key challenge in portfolio construction is to find the asset mix that strikes the right balance between investment risks and expected rewards through retirement. Similarly, the VLCM's custom glide paths are not necessarily the ones that can achieve the highest wealth accumulation (and not even the maximum retirement success metrics), if they come with a portfolio volatility that could be unbearable to plan participants.

Ultimately, retirement success is more likely with a custom portfolio that encourages participants to continue the steady flow of lifetime contributions throughout the normal ups and downs of the markets than with a high return/high risk option that is disconnected from participants' true risk tolerance. Thus, retirement portfolio customization is not about finding the tactical asset allocation that results in the portfolio's outperformance of a benchmark or that can achieve higher wealth accumulation with less savings.

Perhaps the best way to quantify the benefits of glide-path customization to plan participants is through the concept of an "equivalence fee." This refers to the fee (measured in basis points of the assets) that participants would hypothetically be willing to pay in order to be placed in the customized glide path versus staying with the off-the-shelf alternative.

The higher the equivalence fee of a given glide path, the higher the benefit of customization (i.e., the benefit of striking the right risk–return balance) to those plan participants on average. The VLCM calculates the equivalence fee for the custom glide path by using the utility framework (see page 18 in the Appendix for a technical explanation). **Figure 4** displays equivalence fees for custom glide paths resulting from various potential differences in participant populations. Our analysis finds that homogeneous participant populations that significantly differ from the broad population in their risk aversion, their retirement age, or the presence of a DB plan have a relatively higher benefit from glide-path customization.

Figure 4. Quantifying the benefit of customization



Notes: The figure shows the impact of each population characteristic's change from low (25th percentile of broad population data) to medium (50th percentile), as described in Table 2 on page 10.

Source: Vanguard.

IMPORTANT: The projections and other information generated by the VCMM regarding the likelihood of various investment outcomes are hypothetical in nature, do not reflect actual investment results, and are not guarantees of future results. Distribution of return outcomes from VCMM, derived from 10,000 simulations for each asset class and macroeconomic variable modeled. Simulations as of March 31, 2016. Results from the model may vary with each use and over time. For more information, please see the Appendix.

6 A constant 100% allocation to global equities (assuming no leverage) would yield the maximum expected return and expected wealth outcomes over an entire life-cycle in most asset allocation models. Yet most investors with a set time horizon, such as retirement plan participants, would reject such a glide path in favor of a more conservative option.

Glide-path sensitivity

Although the key criteria for glide-path customization are embodied in the fee equivalence, the implication of customization is most visible in the shape of the glide path: average equity exposure, pace, and timing for equity de-risking. Given our modeling assumptions, **Table 1** describes how an increase in each participant population characteristic is expected to affect equity exposure when the other characteristics are similar to the average of the broad population.

The glide-path sensitivities described in Table 1 are intuitive directionally and consistent with standard investment methodology. However, the advantage of building these sensitivities into a framework such as the VLCM is that the model enables quantifying their magnitudes for various population characteristics and ranking them in order of importance. *What characteristics of participant populations matter the most in defining the right glide path for their retirement plans?*

Table 1. Expected impact on glide path

Increase in characteristic	Average equity exposure	Reason
Risk aversion	Decreases	Risk aversion defines participants' attitude toward the retirement portfolio's short-term volatility. More risk aversion (or lower risk tolerance) means participants would rather give up a portion of the portfolio return potential for lower return volatility. Thus, if an investor's risk aversion increases, the equity level of the glide path should decrease.
DB plan income	Increases	DB is a source of income analogous to income from a coupon-bearing bond. The presence of a DB plan increases the share of nonrisky assets in the overall retirement portfolio. Thus, plan participants can afford to take on more equity risk in the custom retirement plan, assuming all other characteristics remain the same.
Savings rate	Decreases	A higher savings rate means faster wealth accumulation through one's working life, meaning that risk-averse participants can afford to de-risk faster and earlier than otherwise (based on our utility assumptions).
Starting salary	Increases or decreases	Assuming that the spending level during retirement will also increase proportionately with starting salary (i.e., constant replacement ratio), the impact on the glide path can go either way. Although a higher salary would lead to faster wealth accumulation and a lower equity allocation, the higher spending requirement would call for a higher equity exposure.
Retirement age	Increases	As retirement age (human capital) increases, salary is earned until a later age, which is analogous to having income from a coupon-bearing bond—and hence plan participants can afford a higher equity exposure in the glide path.
Replacement ratio	Increases	The higher spending requirement would push participants into taking more risk in the glide path as a way to ensure sufficient income.
Correlation of salary to the equity market	Decreases	If labor income is the return on human capital, then a higher correlation with financial market returns would lead to higher risk in the overall portfolio (human + financial capital). Participants may want to offset that higher risk by shifting into lower-risk assets.

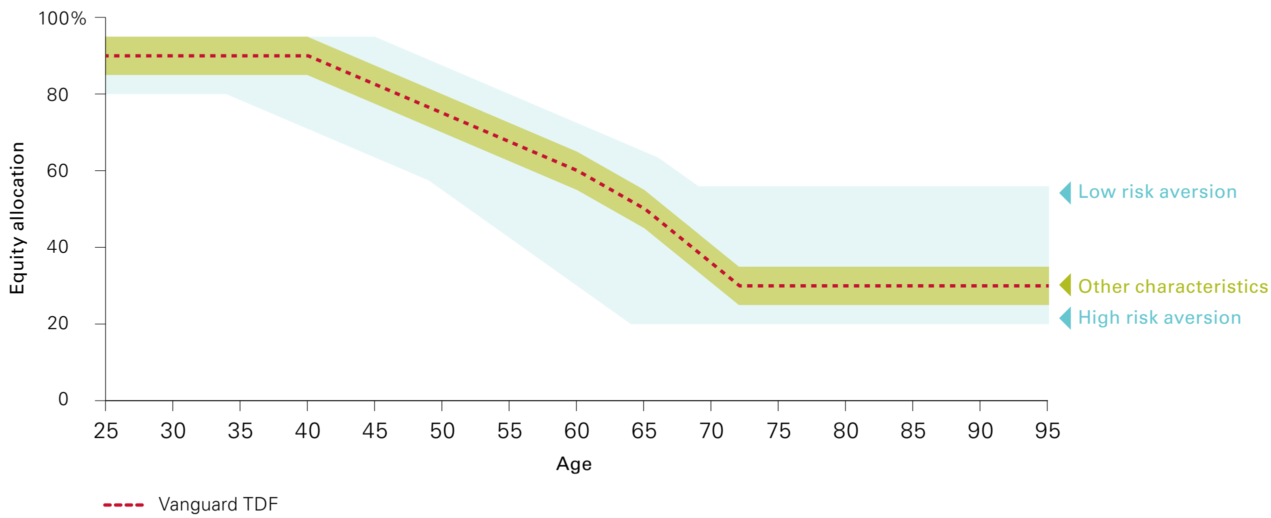
Notes: The table assesses the expected impact if only the specified characteristic changes, rather than multiple characteristics changing simultaneously. As an example, it does not cover a scenario in which the presence of a DB plan (or large initial financial capital) would in turn justify an increase in risk aversion compared with the broad population. In addition, the expected impact on the glide path is based on the utility maximization framework for the VLCM as described in the Appendix. Under other utility and retirement objective functions, the expected impact could vary.

Source: Vanguard.

Figure 5 illustrates the magnitude of the shifts in the model-derived glide paths when considering different degrees of risk aversion across participant populations. The population's risk aversion is the single most important factor behind the glide-path moves. Table 2 on page 10 displays the full ranking of all plan characteristics.

Both Figure 5 and Table 2 indicate that participant populations that are significantly different in average risk aversion, presence of a pension plan, or retirement age have the largest impact on the glide path. On the other hand, customized glide paths for populations with different savings rates, replacement ratios, and salary levels may not meaningfully differ from off-the-shelf solutions.

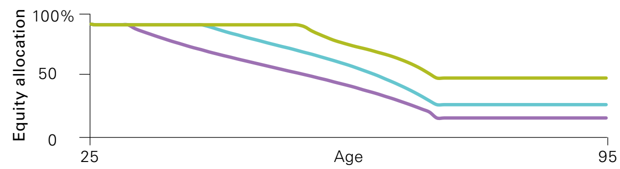
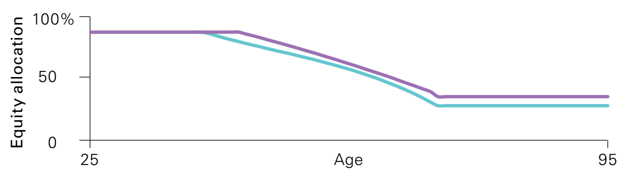
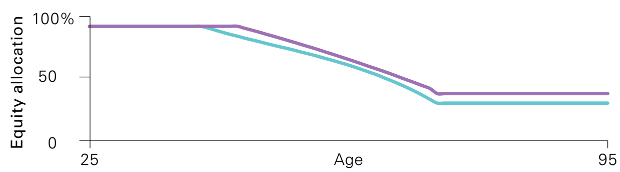
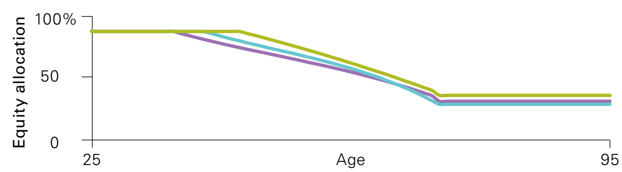
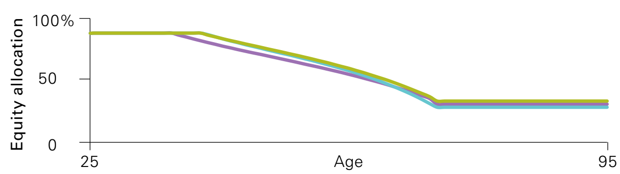
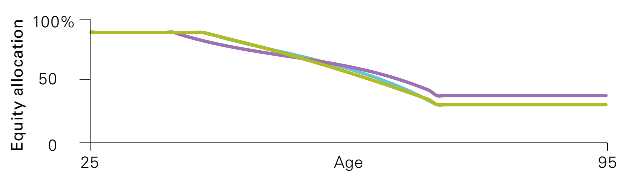
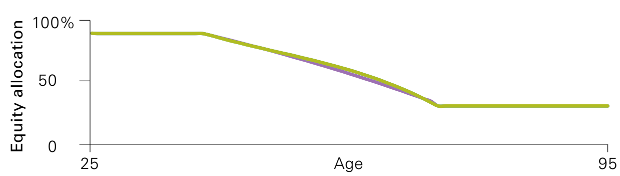
Figure 5. Most population characteristics affect the glide path slightly



Notes: The figure summarizes the impact of each population characteristic changing from low (25th percentile of broad population data) to medium (50th percentile) to high (75th percentile) as described in Table 2 on page 10. VCMM simulations are as of March 31, 2016.

Source: Vanguard.

Table 2: Glide-path sensitivity to changes in population characteristics

Variable	Percentile: Low (25th)	Percentile: Medium (50th)	Percentile: High (75th)	Change in average equity allocation	Effect on shape of model glide paths	Impact
Risk aversion	3.25	4.5	7.0	**12.5%**		
	A measure between 0 and 10 that corresponds to "relative risk aversion."					
Presence of a DB plan	N/A	No DB	Yes (20% RR)	**5.0%**		
	A DB plan functions as a steady additional source of income.					
Retirement age	N/A	65	70	**5.0%**		
	Linked to spending assumptions and is participant population-specific.					
Savings rate	3.5%–7.4%	7.3%–13.0%	12.2%–18.1%	**2.8%**		
	Based on analysis of 1.2 million-plus participants.					
Starting compensation	$30,000	$45,000	$65,000	**1.6%**		
	Based on analysis of 1.2 million-plus participants and research from NACE. Social Security is linked to compensation. Replacement ratio remains unchanged.					
Correlation of wages to markets	–0.5	0	0.5	**1.4%**		
	The degree to which variability in wages is dependent on market returns.					
Replacement ratio (RR)	68%	78%	88%	**0.3%**		
	Percentage of ending salary needed to be replaced by Social Security, TDF, pension plan, etc.					

Notes: Changes in participant population characteristics need to be standardized to compare the impact on glide paths. The low, medium, and high values for characteristics have been standardized to the 25th, 50th, and 75th percentiles of the broad population. For more details, see Table A-1 on page 17 in the Appendix.

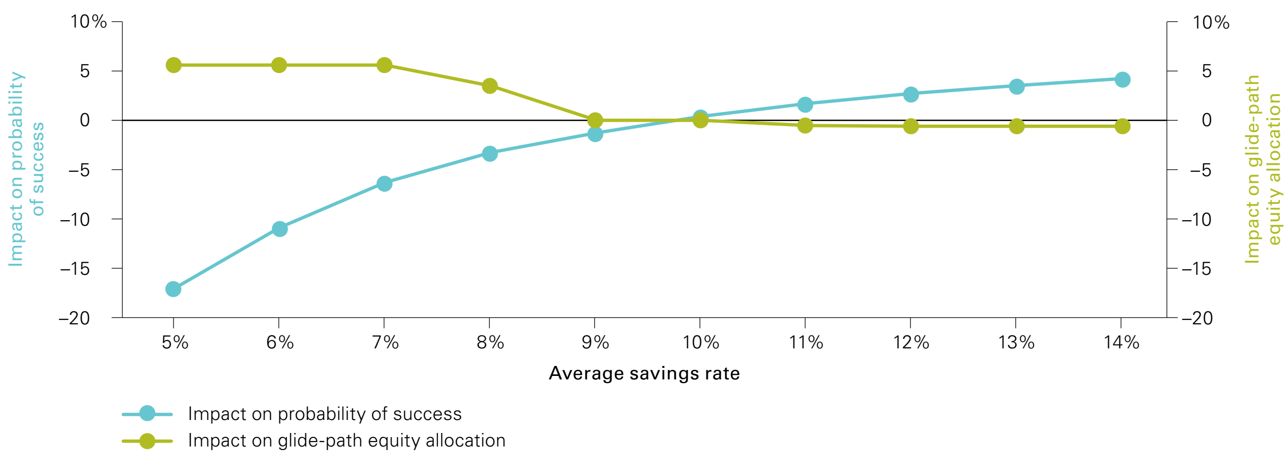
NACE = National Association of Colleges and Employers.

Source: Vanguard.

Also noteworthy is that the glide-path sensitivities appear to be nonlinear in some of the participant population characteristics. **Figure 6** shows a higher glide-path impact from a 1 percentage point increase in contributions at low savings levels (5%–10% contribution rates) than at higher ones. At low savings levels, a 1 percentage point improvement in lifetime retirement savings contributions, along with the power of compounding, has a significant incremental benefit in reducing negative-tail outcomes during retirement (such as shortfall risk, or the odds of depleting wealth). Given this improvement, risk-averse plan participants can afford to move to a lower-risk glide path.

When the savings level is already high, an additional 1% contribution has no significant incremental effect on the probability of success. Thus, the custom glide path need not change much. Through analysis like this, the VLCM can help plan sponsors both to better quantify the importance of encouraging their participants to contribute more, and to anticipate the expected benefit to participants.

Figure 6. The impact of incremental rises in retirement savings contributions is nonlinear



Notes: The figure summarizes the impact of changing the savings rate in 1 percentage point increments when other population characteristics have medium values, as described in Table 2. VCMM simulations are as of March 31, 2016.

Source: Vanguard.

A VLCM case study

The VLCM facilitates several types of savings analysis, including retirement income adequacy and trade-offs among various investment strategies. To illustrate this, let's compare two hypothetical firms—a technology firm and an engineering firm—whose participant populations plan to retire at age 65, have no pension benefits, and have similar salary levels. A key difference is that the technology firm's participants have a low savings rate and low risk aversion, whereas the engineering firm's participants save a large proportion of their salary but have high risk aversion.

At first glance, based on the custom glide paths (see **Figure 7a**), the low risk aversion may appear to make up for the technology firm participants' low savings rate. For additional contrast, comparing custom glide-path outcomes with other alternative glide paths such as Vanguard's TDF and static portfolios (60% equity/ 40% fixed income balanced, 100% equity, and 100% fixed income) gives plan sponsors a valuable frame of reference.

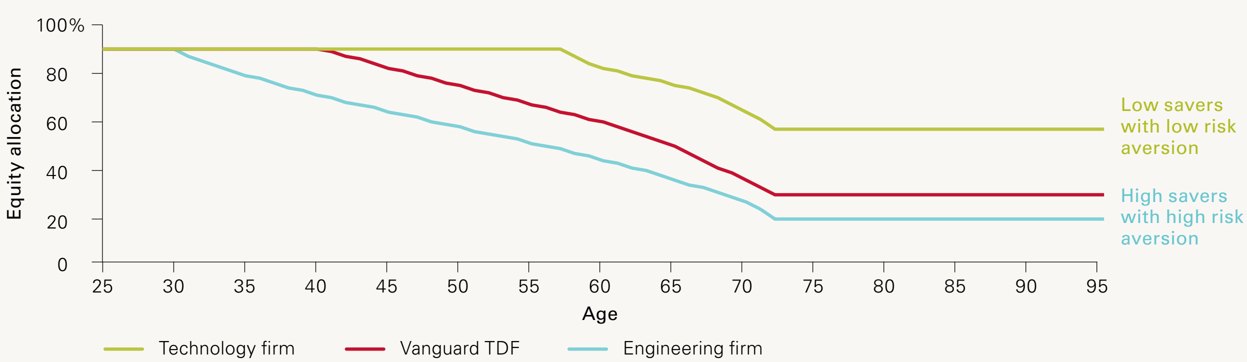
Together, **Figures 7b through 7e** illustrate the risk–return trade-offs of various allocations through glide-path analytics. A closer look at Figures 7b and 7c indicates that the engineering firm participants' higher savings rate results in much higher wealth at retirement compared with the technology firm participants, despite the starkly conservative allocation of the custom glide path. In spite of elevated risk (Figure 7d), a 100% equity portfolio with low savings (Figure 7b) has similar wealth outcomes to the conservative custom TDF with high savings (Figure 7c).

The risk–return metrics can be combined in the context of "probability of success." **Figure 7f** shows that a high savings level leads to a high success rate, whereas low savings translate to a 1-in-5 chance that the participant may have insufficient retirement income. Although much of the industry discussion about customization centers on the glide path's shape or the sub-asset allocation lineup, our analysis shows that solving for those customization factors alone is not enough to overcome inadequate savings levels combined with a simple and disciplined investment approach. These analytics provide an objective context to facilitate a discussion about raising participant savings levels through plan design or education.

The fee equivalence (**Figure 7g**) quantifies the benefit of customization relative to other suboptimal alternatives after accounting for both risk and return. For risk-averse investors like the engineering firm participants in our example, although an all-equity portfolio generates the highest median wealth, the risk–return trade-off is best for the custom glide path, with a benefit (fee equivalence) of 26 basis points per year despite the lower wealth. Conversely, the benefit of customization relative to an all-equity portfolio is much smaller for the more risk-tolerant technology firm participants.

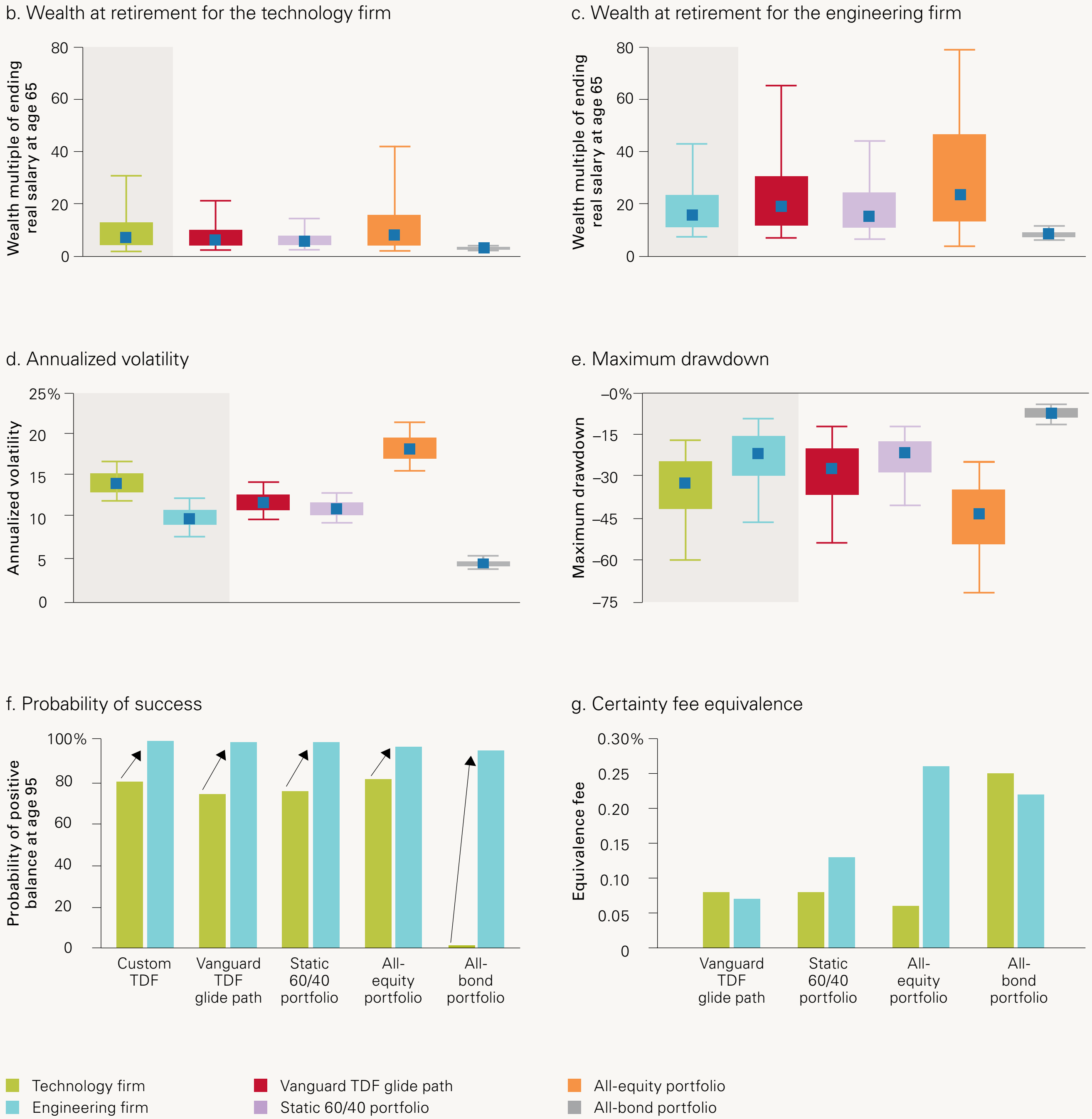
Figure 7. Can an aggressive allocation make up for a low savings rate?

a. Custom glide paths



(Continued on page 13)

Figure 7 *(Continued).* **Can an aggressive allocation make up for a low savings rate?**



Notes: The population characteristics of the technology firm are based on the low values (25th percentile of broad population data) of the average savings rate (5.4%, including employer match), a low risk aversion parameter of 3.25 with no access to a pension plan, and a target retirement age of 65. The engineering firm differs from the technology firm in having a higher average savings rate (14.2%, including employer match) and a higher risk aversion (7.0). Additional details on low, medium, and high values of the broad population are described in Table 2. VCMM simulations are as of March 31, 2016. All the wealth distributions in Figures 7b are statistically different from one another; the same is true for Figure 7c, except for the wealth distribution of the custom glide path and the 60% equity/40% fixed income (60/40) portfolio.

Source: Vanguard.

Sub-asset allocation customization

Another dimension of retirement portfolio customization that draws a lot of attention from plan sponsors involves changes to the sub-asset allocation lineup, including portfolio tilts and sector overweightings and the consideration of alternative asset classes (i.e., commodities, liquid alternatives, and private assets). The VLCM provides a useful framework to consider the quantitative and economic significance of such changes versus other aspects of customization.

Figure 8 compares the impact on the probability of success from certain population characteristics vis-à-vis that from sub-asset class decisions. Clearly, increasing the savings rate from a low level to medium (as indicated in Table 2) dramatically improves the probability of success by about 18%. The importance of plan sponsors educating their participants about raising their savings rate—especially if they see a low savings trend—cannot be overstated. Other factors such as investment costs, delays in retirement, and changes to the replacement ratio have a sizable benefit. By comparison, sub-asset allocation changes (10% of the portfolio weight) such as adding commodities or increasing the credit exposure have a much lower relative impact on the probability of success.

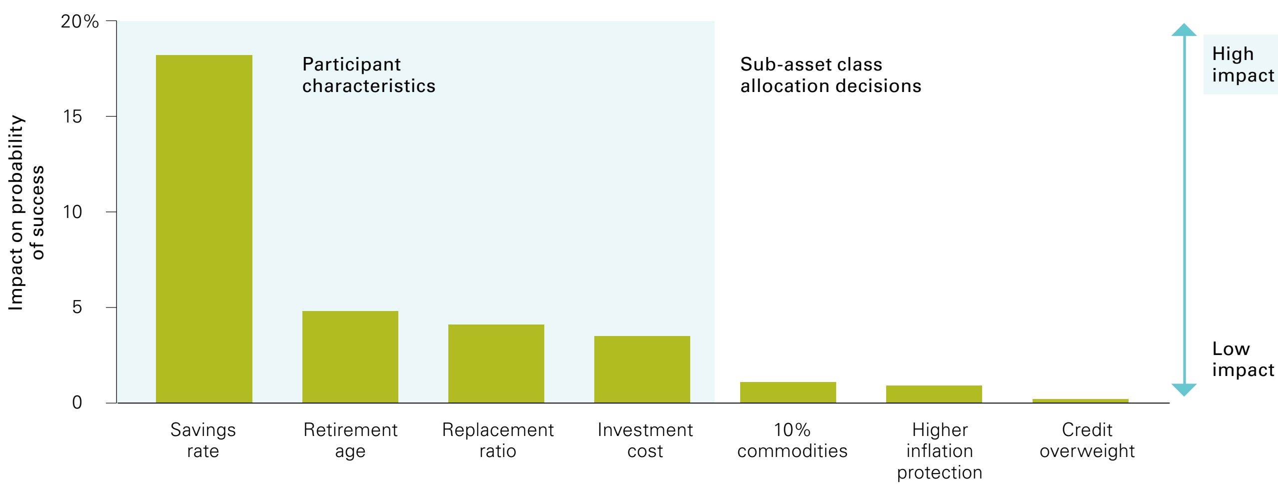
In all fairness, an allocation to 10% commodities could have a significantly better risk–return trade-off (an equivalence fee of 5 basis points per year—as high as for the risk aversion characteristic) while offering only a slight increase in the probability of success.

Caveats of the model

One important limitation of the VLCM is that it does not optimize the levels of spending and contribution rates. Rather, it optimizes the glide path for a given customizable level of spending, growth rate of contributions, and other plan sponsor characteristics. A full dynamic stochastic life-cycle model, including optimization of a savings strategy and dynamic spending in retirement, is beyond the scope of this framework. In addition, the VLCM calculates the wealth distribution at a specific age post-retirement and does not account for age-varying risk aversion. Hence, a plan sponsor assessment of custom glide-path risk may be in order.

Glide-path customization seems reasonable for a relatively homogeneous plan sponsor population that differs from the broad population. Populations with various subpopulations (heterogeneity) can be challenging to customization, as

Figure 8. Sub-asset class allocation decisions



Notes: The figure shows the impact of each population characteristic changing from low (25th percentile of broad population data) to medium (50th percentile) as described in Table 2. VCMM simulations are as of March 31, 2016. Investment cost is the impact on probability of success of a TDF with a fee or investment cost that is 50 basis points higher.

Source: Vanguard.

the glide path is designed for an average participant, and a heterogeneous population may have several participants who differ significantly from the average participant. Hence, a single custom glide path may not be well-suited for all participants in heterogeneous populations.

Conclusion

The Vanguard Life-Cycle Investing Model is a proprietary model created by Vanguard's Investment Strategy Group for glide-path construction customized to average participant population characteristics, to help participants meet their retirement investment goals. The VLCM can benefit plan sponsors by facilitating a deeper understanding of target-date funds in the context of regular due diligence and by enabling glide-path customization through an assessment of risk–return trade-offs among glide paths. The model enables the application of a sound, consistent, and objective process for glide-path construction.

Although much of the industry discussion about customization centers on the shape of the glide path or the sub-asset allocation lineup, our key takeaway is that solving for those factors alone is not enough to overcome an inadequate level of savings.

References

Aon Consulting and Georgia State University, 2008. *Replacement Ratio Study: A Measurement Tool for Retirement Planning*. Chicago: Aon Consulting.

Arnott, Robert, 2012. *The Glidepath Illusion*. Research Affiliates Fundamentals (September), 1–5.

Arnott, Robert D., Katrina F. Sherrerd, and Lillian Wu, 2013. The Glidepath Illusion … and Potential Solutions. *The Journal of Retirement* 1(2), 13–28.

Basu, Anup K., and Michael E. Drew, 2009. Portfolio Size Effect in Retirement Accounts: What Does It Imply for Lifecycle Asset Allocation Funds? *Journal of Portfolio Management* 35(3): 61.

Basu, Anup K., Brett Doran, and Michael E. Drew, 2012. *Sequencing Risk: A Key Challenge to Creating Sustainable Retirement Income*. Financial Services Institute of Australasia, Research Report (October), 1–40.

Bodie, Zvi, Robert C. Merton, and William F. Samuelson, 1992. Labor Supply Flexibility and Portfolio Choice in a Life-Cycle Model. *Journal of Economic Dynamics and Control* 16(3–4): 427–449.

Brancato, Matthew, Stephen P. Utkus, John Schadl, and Catherine Gordon, 2014. *Perspectives on Custom Target-Date Strategies in DC Plans*. Valley Forge, Pa.: The Vanguard Group.

Davis, Joseph, Roger Aliaga-Díaz, Harshdeep Ahluwalia, Frank Polanco, and Christos Tasopoulos, 2014. *Vanguard Global Capital Markets Model*. Valley Forge, Pa.: The Vanguard Group.

Donaldson, Scott J., Francis M. Kinniry Jr., Vytautas Maciulis, Andrew J. Patterson, and Michael A. DiJoseph, 2015. *Vanguard's Approach to Target-Date Funds*. Valley Forge, Pa.: The Vanguard Group.

Estrada, Javier, 2016. The Retirement Glidepath: An International Perspective. *The Journal of Investing* 25(2): 28–54.

Gomes, Francisco J., Laurence J. Kotlikoff, and Luis M. Viceira, 2008. Optimal Life-Cycle Investing with Flexible Labor Supply: A Welfare Analysis of Life-Cycle Funds. A*merican Economic Review* 98(2): 297–303.

Pfau, Wade D., and Michael E. Kitces, 2014. Reducing Retirement Risk with a Rising Equity Glide Path. *Journal of Financial Planning* 27(1): 38–45.

Shiller, Robert J., 2005. *Life-Cycle Portfolios as Government Policy*. The Economists' Voice 2(1): 1–14.

U.S. Department of Labor, Employee Benefits Security Administration, 2013. *Target Date Retirement Funds–Tips for ERISA Plan Fiduciaries* (February); available at https://www.dol.gov/ebsa/newsroom/fstdf.html.

Utkus, Stephen P., and Jean A. Young, 2004. *Lessons from Behavioral Finance and the Autopilot 401(k) Plan*. Valley Forge, Pa.: The Vanguard Group.

Vanguard Group, The, 2014. *How America Saves 2014*. Valley Forge, Pa.: The Vanguard Group.

Appendix I. About the Vanguard Capital Markets Model

IMPORTANT: The projections or other information generated by the Vanguard Capital Markets Model regarding the likelihood of various investment outcomes are hypothetical in nature, do not reflect actual investment results, and are not guarantees of future results. VCMM results will vary with each use and over time.

The VCMM projections are based on a statistical analysis of historical data. Future returns may behave differently from the historical patterns captured in the VCMM. More important, the VCMM may be underestimating extreme negative scenarios unobserved in the historical period on which the model estimation is based.

The VCMM is a proprietary financial simulation tool developed and maintained by Vanguard's Investment Strategy Group. The model forecasts distributions of future returns for a wide array of broad asset classes. Those asset classes include U.S. and international equity markets, several maturities of the U.S. Treasury and corporate fixed income markets, international fixed income markets, U.S. money markets, commodities, and certain alternative investment strategies. The theoretical and empirical foundation for the Vanguard Capital Markets Model is that the returns of various asset classes reflect the compensation investors require for bearing different types of systematic risk (beta). At the core of the model are estimates of the dynamic statistical relationship between risk factors and asset returns, obtained from statistical analysis based on available monthly financial and economic data. Using a system of estimated equations, the model then applies a Monte Carlo simulation method to project the estimated interrelationships among risk factors and asset classes as well as uncertainty and randomness over time. The model generates a large set of simulated outcomes for each asset class over several time horizons. Forecasts are obtained by computing measures of central tendency in these simulations. Results produced by the tool will vary with each use and over time.

The primary value of the VCMM is in its application to analyzing potential client portfolios including ones with glide paths. VCMM asset-class forecasts—comprising distributions of expected returns, volatilities, and correlations—are key to the evaluation of potential downside risks, various risk–return trade-offs, and the diversification benefits of various asset classes. Although central tendencies are generated in any return distribution, Vanguard stresses that focusing on the full range of potential outcomes for the assets considered, such as the data presented in this paper, is the most effective way to use VCMM output. We encourage readers interested in more details to read Davis et al. (2014).

The VCMM seeks to represent the uncertainty in the forecast by generating a wide range of potential outcomes. It is important to recognize that the VCMM does not impose "normality" on the return distributions, but rather is influenced by the so-called fat tails and skewness in the empirical distribution of modeled asset-class returns. Within the range of outcomes, individual experiences can be quite different, underscoring the varied nature of potential future paths. Indeed, this is a key reason why we approach asset-return outlooks in a distributional framework.

Appendix II. Participant population characteristics

Several characteristics of the participant population are needed to model wealth outcomes in the VLCM. Details of the characteristics of the broad population are listed in Table A-1 to provide a benchmark for comparison.

Table A-1. Custom characteristics of a participant population

Characteristic	Description
Risk aversion	This can be defined as aversion to uncertainty of outcomes. In other words, a risk-averse investor prefers a degree of certainty. An investor with low risk aversion tolerates the uncertainty for a better outcome, and a risk-neutral investor cares only about the best outcome and is indifferent toward uncertainty.
DB plan income	This is computed by: • *Defined benefit replacement ratio:* the typical proportion of a final-year salary that a defined benefit will cover, or • *Defined benefit formula:* the calculation used by the plan sponsor to contribute to a defined benefit plan.
Savings rate	Rates typically differ by age, with investors saving relatively less when young and relatively more as they approach retirement. Based on information from *How America Saves 2014*, the average investor starts at age 25 with a 5% savings rate and ends at age 65 with a 10% savings rate.
Starting salary	The median starting salary (capped at $260,000) for the full-time employee population, typically at ages 23–27.
Retirement age	The average age at which plan participants leave the workforce.
Starting age	The average starting age of participants.
Starting balance	The median starting balance in the plan. This variable is useful when a typical employee starts at a firm well after entering the workforce and has significant tax-deferred savings.
Human capital riskiness	The volatility and correlation of wages relative to the financial assets that investors own can affect the slope of the glide path. The plan sponsor's industry from the following list can help determine human-capital riskiness: • Construction • Education and health services • Financial activities • Information • Leisure and hospitality • Manufacturing • Natural resources and mining • Other services • Professional and business services • Retail trade • Utilities • Transportation and warehousing • Wholesale trade
Social Security	A U.S. government program that provides income to retirees and encompasses several other social welfare and insurance programs.

(Continued on page 18)

Table A-1 *(Continued).* Custom characteristics of a participant population

Characteristic	Description
Replacement ratio (spending requirement)	We follow industry convention in assuming that retirees will spend a percentage of their ending salary in retirement from a combination of Social Security benefits and investment income from private sources. The replacement ratio assumption is consistent with retirees maintaining the same standard of living as during their final working years. Replacement ratios vary by income level, as Social Security makes up a smaller percentage at larger salaries. To assign replacement ratios, Vanguard draws on work by Aon Consulting, with data from the Consumer Expenditure Survey produced by the U.S. Department of Labor's Bureau of Labor Statistics.
Wage scale	Investor salary growth is modeled after the U.S. Social Security Administration's wage index. That index is based on reported wages among workers aged 25–65 for low, medium, and high earners. This wage scale allows us to trace the earnings progression of an average earner over a 40-year career. We allow for an additional serial increase in real wages to account for economic productivity growth.
Correlation of wages to market returns	Wage growth correlation with market returns can be incorporated into the model to assess the impact on wealth outcomes and glide-path design.

Appendix III. Utility maximization and equivalence fee

The optimal glide path is a solution to the following life-cycle investing problem:

$$\underset{\{\propto_t\}}{Max} \; E_0[U(W_T)]$$

$$s.t. \;\; 0 < \propto_t \leq 1$$

$$W_T = \sum_{t=0}^{T} (Savings_t - Spending_t) \prod_{j=t}^{T} (1+R_j^p) + \sum_{t=x}^{T} \frac{SS_t M}{(1+r_t)^{t-x}} + \sum_{t=x}^{T} \frac{DB_t M}{(1+r_t^c)^{t-x}}$$

$$R_t^p = \propto_t r_t^s + (1-\propto_t) r_t^b$$

Where W_T is total wealth post-retirement; $\propto$ is equity allocation in the glide path over time; R^p, r^s, and r^b are portfolio and asset class returns, respectively, from VCMM projections; SS_t is Social Security payments; M is the mortality rate; DB_t is defined benefit plan payments; x is the retirement age; r_t is the risk-free discount rate at time t; and r_t^c is the corporate-grade discount rate at time t.

$U(W_T)$ is the utility function for a risk-aversion level (Υ) given by:

$$U(W_T) = \frac{W_t^{1-\Upsilon}}{1-\Upsilon}$$

The equivalence fee calculation finds a fee, $\boldsymbol{\varphi}$, such that the following is true:

$$E_0[U(W_T^*)] = E_0[U(W_T')]$$

Where

$$W_T^* = f(1+R_t^{p*})$$

$$W_T' = f(1+R_t^{p'} - \boldsymbol{\varphi})$$

Where R_t^{p*} is the off-the-shelf TDF portfolio return and $R_t^{p'}$ is the return of the optimal glide path.

Connect with Vanguard® > vanguard.com

For more information about Vanguard funds, visit vanguard.com or call 800-662-2739 to obtain a prospectus or, if available, a summary prospectus. Investment objectives, risks, charges, expenses, and other important information about a fund are contained in the prospectus; read and consider it carefully before investing.

CFA® is a registered trademark owned by CFA Institute.

Vanguard Research

P.O. Box 2600
Valley Forge, PA 19482-2600

ISGLCIM 092016