Making the implicit explicit: A framework for the active-passive decision

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- How should an investor allocate across active and passive investments? It's a challenging decision with many components. In the absence of a structured decision-making process, investors are left making arbitrary decisions based on implicit assumptions.

- In this paper, we provide a quantitative framework for active-passive decision-making and aim to shed light on those implicit assumptions by highlighting the explicit attributes affecting the process. We employ a model using four key variables—gross alpha expectation, cost, active risk, and active risk tolerance—to establish active and passive investment allocation targets for a range of investor types.

- Indexing is a valuable starting point for all investors, and many may index their entire portfolio. But our analysis shows that for those who are comfortable with the characteristics of active investments, an allocation to active can also be a viable solution.

Introduction

Imagine for a moment that you live in a world with only two fund options: a passively managed fund[1] and an actively managed fund[2] with similar levels of volatility. You, the investor, are trying to determine how to structure your two-fund portfolio.

The expected relative return of the active fund is simply a function of two variables—gross alpha expectation and cost. If the resulting net relative performance (gross alpha minus cost) is expected to be positive, the simple choice would be to allocate 100% to the active fund. If, on the other hand, the resulting net alpha of the active fund is expected to be negative, the choice would be equally straightforward—allocate 100% to the passive fund. This approach results in a binary choice—either all active or all passive.

It is this dynamic that is often at the heart of the active-passive debate, which tends to focus on all-or-nothing views and recommendations. Proponents of passive investing point to research demonstrating that the median active manager underperforms after costs and that outperformers are difficult to recognize in advance. Meanwhile, proponents of active investing argue that despite the underperformance of the median active fund, many active managers do still add value, and the impact of possible outperformance can be significant. And so the debate rages on.

We reject this basic, binary choice. Both active and passive investments have potential benefits in a portfolio. Passive funds offer low-cost benchmark tracking, leading to a tight range of relative returns. Active funds offer the potential for outperformance in exchange for a wider range of relative returns (in other words, greater uncertainty) and typically higher costs.[3]

With this in mind, let's return to our original thought exercise. But this time, in addition to gross alpha and cost, let's consider two more variables: active risk (defined as the uncertainty of future manager performance) and active risk tolerance (the degree to which an investor can tolerate this uncertainty).[4] Now we can consider a more nuanced trade-off between active and passive by incorporating an "uncertainty penalty" to our active expectations. This can help balance the potential positive impact of alpha expectations with the uncertainty of achieving a favorable outcome.

We can then incorporate more details to help make our decision. For example, would it be prudent to invest in the active fund if it is expected to provide 0.10% net annualized outperformance?

Some degree of uncertainty is inherent in any active decision. Despite a possible positive relative return expectation, there is still a chance that the manager won't achieve the expected outperformance. In this case, the modest size of the potential reward may not be substantial enough to justify a 100% allocation to the active fund given its uncertainty.

But what about a 5% allocation to this fund as part of an active-passive portfolio? What about 25%? How does the level of active risk inherent in the fund affect this decision? How does your own (or your organization's, or your client's) tolerance for taking on active risk affect it? And what if we increase or decrease the gross alpha expectation or cost associated with investing in the fund? These are the types of questions we consider in this paper.

We aim to assist the active-passive decision-making process by enabling investors to think more deliberately about their expectations and the risks they're willing to accept. Our framework makes these expectations explicit, a valuable contribution to ongoing due diligence and regular calibration of the conditions that justify a given active-passive mix.

Notes on risk

Please remember that all investments involve some risk. Be aware that fluctuations in the financial markets and other factors may cause declines in the value of your account. There is no guarantee that any particular asset allocation or mix of funds will meet your investment objectives or provide you with a given level of income.

All investing is subject to risk, including possible loss of principal.

1 The passive fund in the context of our paper is a market-cap-weighted index fund in a single asset class—for example, a broad-based U.S. equity index fund.

2 The active fund in the context of our paper is a traditional actively managed fund in the same sub-asset class as the passive fund—for example, a U.S. equity active fund that uses bottom-up security selection.

3 Active funds, on average, tend to have higher total expense ratios as well as higher tax costs for those subject to tax. See Philips, Kinniry, Walker, Schlanger, and Hirt (2015).

4 Active risk tolerance is defined as the willingness with which an investor would take on active risk in return for an uncertain benefit of positive alpha. We'll talk in more detail later about how this influences the active-passive decision-making process.

The active-passive decision framework

The portfolio construction process begins with establishing an appropriate strategic asset allocation. A secondary but important decision is how to implement the asset class and sub-asset class exposures determined in the first step. It is at this point that specific investment products are evaluated and the decision to allocate between active and passive investments will be made. See **Figure 1** for an illustration of the hierarchy of portfolio decisions.

This paper offers a framework to enable investors to more explicitly approach and evaluate the mix of active and passive investments in their portfolios. It identifies the key decision factors all investors are subject to when determining a reasonable balance based on their individual circumstances.[5] It does not purport to promise better returns but rather to offer a clear decision-making process investors can use to establish a target allocation. This framework can also be used with other strategies such as factor investing. See **Appendix B** for more detail.

Prior Vanguard research has affirmed the active-passive decision as a broad strategic decision rather than a regional or time period-specific one. This differs from the belief that active funds can best flourish in specific market segments or time periods. We find that neither the market segments nor the time of the market cycle ensures better performance. Instead, active management requires talent, low costs, and patience to prosper.[6]

Our framework considers the impact of four variables related to these tenets of active management success:

- Gross alpha expectation
- Cost
- Active risk
- Active risk tolerance.

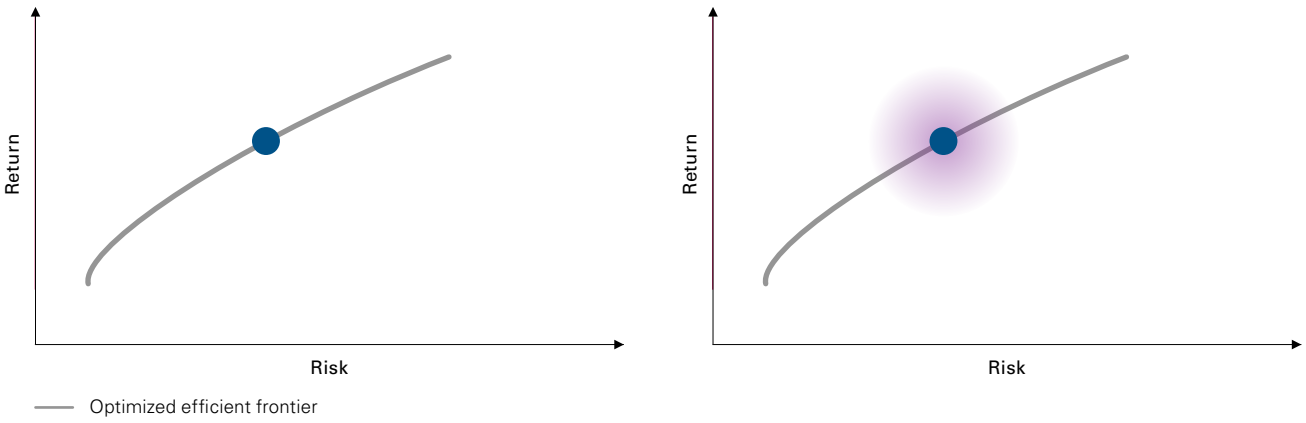
Gross alpha expectation: A judgment about talent

Gross alpha expectation is the anticipation of one's ability to achieve successful outcomes. An investor's degree of gross alpha expectation about his or her active manager selection skill is a critical component of the active allocation decision. It is important to note that the expectation of alpha does not necessarily translate into actual alpha—not all decision-makers can be above average (Sharpe, 1991). Because behavioral biases such as overconfidence can lead to unreasonable expectations, a realistic assessment is critical.

Figure 1. Active-passive decision-making during portfolio construction

a. Select strategic asset allocation based on asset class expectations (blue dot)

b. Adding active funds adds manager risk. It introduces idiosyncratic outcomes that could increase or decrease both the return and risk of the portfolio (purple shading).



Note: Illustrations represent a hypothetical efficient frontier based on asset class expectations and do not represent a particular investment.
Source: Vanguard.

5 See Baks, Metrick, and Wachter (2001), Waring et al. (2000), and Waring and Siegel (2003) for prior research on the active-passive allocation decision process and methods of addressing investors' underlying assumptions.

6 For a review of the empirical work done on the lack of systematic outperformance by investment type, see Davis et al. (2007). For research regarding market periods' lack of impact on active performance, see Philips (2008) and Philips, Kinniry, and Walker (2014). For a discussion of what does affect active management success, see Wallick, Wimmer, and Balsamo (2015a).

Each investor will have his or her own methods of attempting to identify talented managers and developing a gross alpha expectation for them.[7] This is typically best done through a rigorous due-diligence process combined with an understanding of alpha ranges and sensitivity to the probability of success.

The level of expected alpha is a subjective measurement; actual future alpha levels are uncertain. In our framework (as discussed further below), the term "alpha expectation" carries a statistical meaning; the manager assessment can be thought of in terms of a distribution or bell curve of potential alpha outcomes (see **Figure 2**). The central tendency or mean of the distribution is the expected alpha, and its standard deviation is a function of the manager's active risk.

Cost: The enemy of net alpha

Evidence shows that the odds of outperformance increase as investors are able to reduce the cost of investing in active strategies.[8] Indeed, low cost is the most effective quantitative factor that investors can use to improve their chances of success.[9] The cost of an active fund is also much more predictable than gross alpha. Gross alpha expectation and cost combine to form the net alpha expectation.

Active risk: Uncertainty quantified

Any active fund by its nature deviates from a benchmark in the attempt to improve returns. No active manager will outperform the market every day, every week, every month, or even every year. Even those managers who have provided successful performance over longer time frames have typically experienced extended periods of underperformance.[10] This inconsistent pattern of relative returns can be quantified as active risk (i.e., tracking error), or the volatility of a fund relative to its target benchmark, and can be thought of as the uncertainty the investor attaches to that particular active manager. This is not to assume that higher tracking error necessarily leads to higher returns,[11] but rather that active funds may take on a range of different tracking errors.[12]

When compounded over time through the holding period of the investment, active risk leads to variation in performance outcomes that can differ substantially from the central gross alpha expectation for a manager. In other words, active risk and gross alpha expectations both have a straightforward statistical interpretation in terms of the standard deviation and mean derived from the bell curve of potential performance outcomes (see Figure 2).

This distributional interpretation of active manager skill has been missing in the traditional active-passive debate, in which a manager's alpha is typically thought of in terms of a point forecast. Incorporation of this distribution is the distinctive feature of our framework.

Active risk tolerance: A proxy for patience

The final element in evaluating the potential use of active strategies is the degree to which an investor is willing to take on active risk in the pursuit of outperformance. At the heart of the active-passive framework is a trade-off between an investor's subjective alpha expectation and his or her subjective tolerance for downside risk, as depicted in Figure 2. The active-passive decision arises from balancing the two.

How the variables affect active-passive allocation

Under this interpretation of active-passive allocation as the solution to the active risk-return trade-off, one can think of indexing as a diversifier of active manager risk. Investors uncomfortable about assuming the full

7 Vanguard's experience with active managers has found that successful identification of future gross alpha is not based on past performance. Instead, it is more reasonably based on highly qualitative assessments of the manager's people, process, philosophy, and firm. For more on Vanguard's approach to selecting managers, see Wallick, Wimmer, and Balsamo (2015a) and Wallick, Wimmer, and Martielli (2013).

8 See Wallick, Wimmer, and Balsamo (2015b) for further details.

9 For the purposes of this analysis we have assumed indexing is low-cost. Although not all index funds are low-cost, many market-cap-weighted index funds are. In addition, although active funds are, on average, higher-cost than the average index fund, not all active funds are high-cost, and so we consider a range of expense ratios.

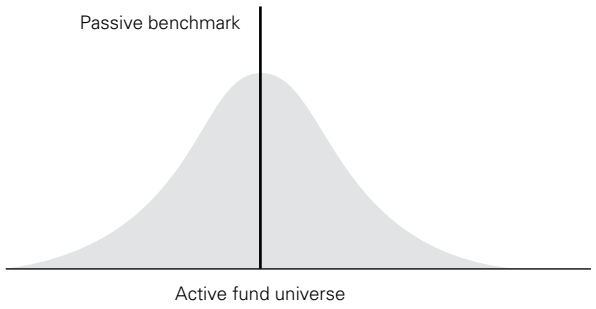
10 Previous Vanguard research has discussed patience as one of the keys to successful use of active management. Active risk tolerance can also be thought of as how much patience an investor exhibits regarding fund volatility relative to the benchmark over time. See Wimmer, Chhabra, and Wallick (2013) for further discussion of successful active managers' patterns of returns.

11 See Schlanger, Philips, and LaBarge (2012) for further discussion of this point.

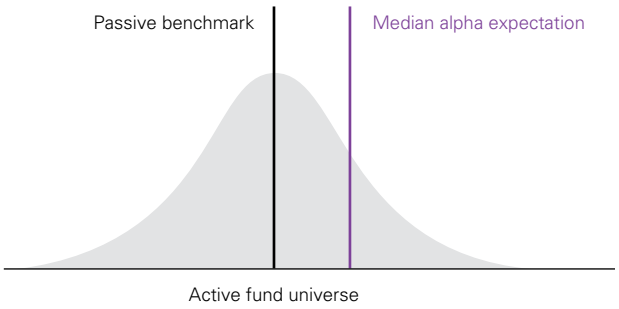
12 This paper uses active risk as the key differentiating characteristic between fund strategies. History indicates that fund tracking errors have typically stayed within reasonable bands over time, thus providing a useful barometer for future expectations. Active share could also have been used to measure funds' levels of relative risk, or "activeness." Although tracking error and active share are different measures, Vanguard research indicates that they are quantitatively linked. We chose to use tracking error because better data are available and investors are more widely aware of how it's calculated.

Figure 2. Alpha expectation is a median surrounded by a range of possible outcomes

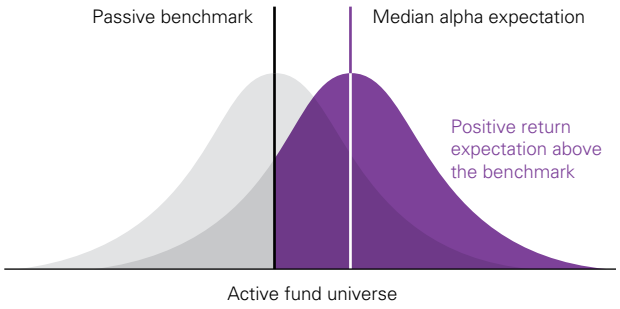
a. Performance probability distribution for a randomly chosen active manager



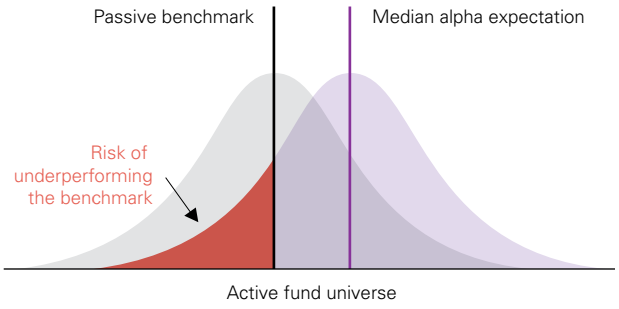
b. Investor's (subjective) performance expectation for a given active manager



c. Returns can span a variety of possible outcomes



d. Active manager risk and the possibility of underperfomance must be considered



Note: The pale and dark purple areas represent a hypothetical alpha distribution for a randomly chosen active manager for which the investor has a positive alpha expectation. The red area represents the hypothetical risk of underperforming the benchmark.

Source: Vanguard.

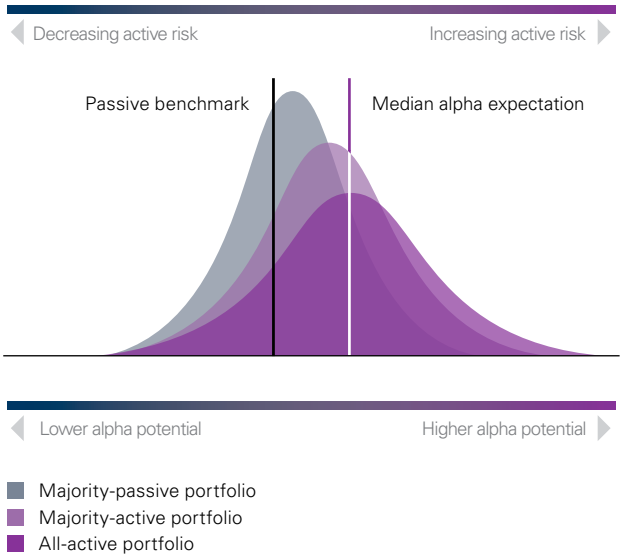
amount of active risk associated with a given manager can mitigate the uncertainty by adding more of the "active-risk-free" asset to the portfolio (see **Figure 3**). However, as they do so, any alpha expectation they had for that manager will also be diluted (i.e., as shown in Figure 3, the various distributions of portfolio outcomes become narrower and shift to the left). The correct allocation is the one that strikes the right balance between risk and expected active reward. The implication of Figure 3 is that even when an investor attaches a positive alpha expectation to a manager, adding some indexing to the mix can mitigate manager risk.

Before moving on to the quantitative application of this approach, it's helpful to consider at a high level how underlying assumptions for each of our four variables would influence the allocation, as shown in **Figure 4**.

Remember that these factors are to be evaluated in regard to their impact on the active selections being considered, not the passive alternatives (we know passive funds have a gross alpha expectation of zero, can be obtained at a very low cost, and offer relatively little active risk). The attributes can be thought of in terms of a sliding scale, each one leading an investor to lean more toward active or passive.

Were we to stop here, we would be left with a completely qualitative allocation process. As described so far, the process outlines the importance of each factor, but it is incomplete if our goal is to be explicit about how to weigh one characteristic relative to another. Indeed, any final decision using solely this approach would be arbitrarily based on implied assumptions. To avoid this, we have constructed a quantitative model that can consider different levels of each of the decision factors above and better tailor solutions to specific circumstances.

Figure 3. Indexing can reduce active manager risk



Source: Vanguard.

Figure 4: Key decision factors and their impact on the active-passive mix

Source: Vanguard.

From qualitative to quantitative: A three-step process

Our quantitative simulation framework consists of three steps:

1. Build simulations for active managers.
2. Calculate the distribution of potential manager outcomes, including the associated underperformance risk.
3. Solve for the active-passive allocation that strikes the right balance between active risk and expected net alpha.

This framework provides investors with tailored active-passive allocation targets based on their inputs and preferences. Next, we'll describe each component of the framework and how the three steps lead to the target allocation.

A quantitative framework for active-passive decisions

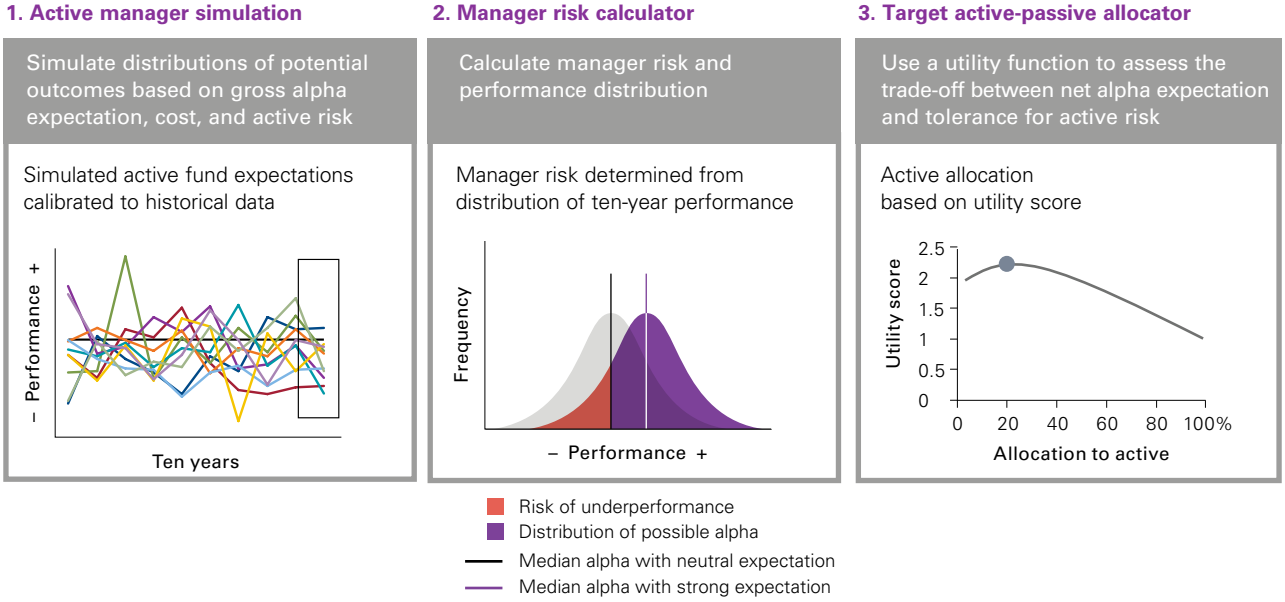
The simulation model shown in **Figure 5** consists of three linked components: an active fund universe simulation, a manager risk calculation, and a risk-return optimization to find the allocation that best suits the investor's attitude toward active risk.

Active manager simulation

The first component, the active manager simulation, creates a theoretical universe of active mutual fund outcomes based on inputs of gross alpha expectation, cost, and active risk. Each combination of these three variables is used to generate Monte-Carlo simulations of 10,000 different possible performance paths over a ten-year period.

We effectively create 45 universes of 10,000 funds for 45 different investor scenarios: 5 levels of gross alpha expectation x 3 levels of cost x 3 levels of active risk = 45 scenarios. These levels and their definitions are discussed in more detail below.

Figure 5. The quantitative process



Gross alpha expectations of the active fund(s):
Very low, low, neutral, high, very high

Cost (expense ratio) of the fund(s):
Lower, moderate, higher

Active risk (tracking error) of the active fund(s):
Lower, moderate, higher

Manager risk:
10,000 active manager simulations create a distribution of performance around alpha expectation

Risk tolerance:
A utililty function solves the risk-return trade-off for a given level of risk tolerance

The Monte-Carlo method is a computational technique that uses random sampling to calculate a number of future scenarios. We generally sample from historical data or from a user-defined probability distribution. The potential outcomes become part of a series of simulations to help summarize a distribution of results.

Source: Vanguard.

Performance distribution and manager risk

The second component of the model is the manager risk calculation. It compiles the distributions of each hypothetical 10,000-fund universe in order to calculate the range of uncertainty of the funds' performance.

A simulated distribution of managers with a median net alpha expectation of zero is illustrated by the bell curve around the gray line in the center of Figure 5. A sample distribution of managers with a positive median net alpha expectation is illustrated by the curve around the purple line. Although the median for these 10,000 funds is positive, uncertainty remains, as evidenced by the range of possible outcomes and the sizable tail risk of those with a negative net alpha.

Target active-passive allocation

The third component of the quantitative framework is a utility-function-based calculation. It uses active risk tolerance to assess the trade-offs between the active portfolio (represented by a distribution based on net alpha expectations and active risk) and the passive portfolio (represented by a single outcome—not a distribution—based on its relatively low level of active risk). This is completed for each of the 45 investor scenarios across 3 different active risk tolerance levels, for a total of 135 active-passive allocations.

Utility-adjusted wealth creation over the simulated ten-year period is calculated for a full range of active-passive combinations. The suggested allocation is the one that maximizes utility-adjusted wealth in each of the 135 scenarios. The mathematical process is detailed in Appendix C. **Figure 6** illustrates the sequence of decisions an investor needs to make when considering an active-passive mix.

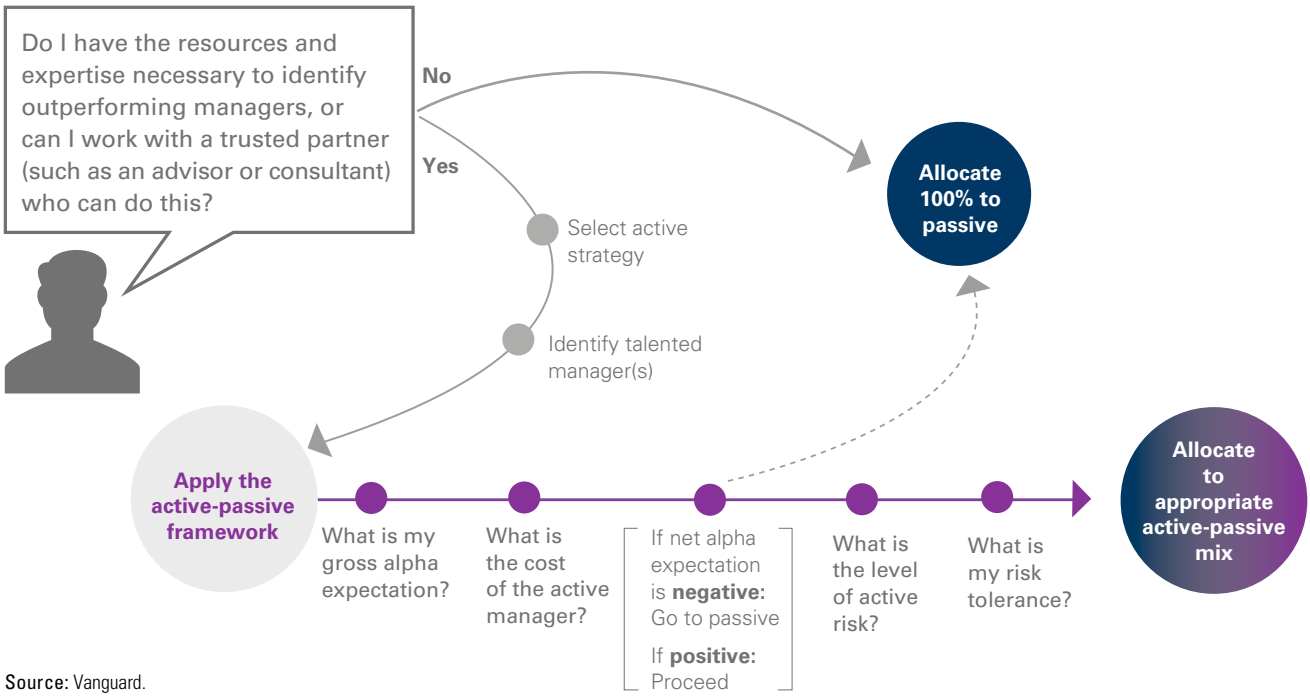
Calibrating the simulation parameters

As with all forward-simulation models, our active-passive calculations are not an assessment of historical probabilities. Instead we have developed a prospective framework for decision-making. The range of figures in each of our scenarios is informed by history but not reflective of any one particular history.

History is only one of many possible outcomes, and selective histories will provide different results. We illustrate a general approach to parameter calibration with an analysis of U.S. active mutual fund data. We used asset-weighted, factor-adjusted U.S. active mutual fund data for the ten years ended June 30, 2016, to calibrate the ranges for our parameters (five levels of gross alpha expectations, three levels of cost, three levels of active risk, and three levels of active risk tolerance).

Two of these parameters—gross alpha expectation (gross alpha) and active risk (tracking error)—are illustrated in **Figure 7**, a visualization of the historical data used to form the range of inputs used in our case study on page 9.[13]

Figure 6. Active-passive decision flowchart



Source: Vanguard.

13 See Appendix A for details on how many funds meet each set of gross alpha and active risk criteria.

Applying the framework to an equity portfolio: A case study

To further demonstrate how the quantitative approach can be applied in practice, from here on we will discuss the framework in the context of an investor determining a U.S. equity allocation. Although we focus here on one asset class—U.S. equities—a similar approach could be applied to a wide range of asset classes.

We conducted our analysis on two levels. Part one includes gross alpha expectations and cost but excludes a consideration of how active risk and active risk tolerance affect the results. Part two accounts for the uncertainty of active manager performance and varying levels of investor tolerance to that uncertainty.

Part one analysis: Gross alpha expectation and cost

To determine gross alpha expectation—the expectation of selecting outperforming active managers—we subdivided our simulation's population into five skill levels: very low, low, neutral, high, and very high. We then quantified what portion each skill level represented and how these segments calibrated to recent historical data:

- **Very low gross alpha expectation:** an expected random selection from the bottom one-third of the entire active manager population (which has a median annualized gross alpha of –1.28%).
- **Low gross alpha expectation:** an expected random selection from the bottom two-thirds of the entire active manager population (which has a median annualized gross alpha of –0.42%).
- **Neutral gross alpha expectation:** an expected random selection from the entire active manager population (which has a median annualized gross alpha of 0.16%).
- **High gross alpha expectation:** an expected selection from the top two-thirds of the entire active manager population (which has a median annualized gross alpha of 0.85%).
- **Very high gross alpha expectation:** an expected selection from the top one-third of the entire active manager population (which has a median annualized gross alpha of 1.54%).

Figure 7. Gross alpha expectations and active risk

Notes: Data are for the ten-year period from July 1, 2006, through June 30, 2016, and represent active equity funds with at least 36 months of history available to U.S. investors in the following categories: small-cap value, small-cap growth, small-cap blend, mid-cap value, mid-cap growth, mid-cap blend, large-cap value, large-cap growth, and large-cap blend. Funds that died or merged were included in the analysis. The oldest and lowest-cost single share class was used to represent a fund when multiple share classes existed. Asset-weighted results were calculated using each fund's average reported monthly assets. Each fund is represented one time in the figure; because the analysis is asset-weighted, the median gross alpha and median tracking error will not lie in the middle of the ranges for each alpha level and tracking error in the chart above. Alpha was calculated by regressing monthly gross returns against the Fama-French three factors of small minus big, high minus low, and excess return on the market over the risk-free rate. Tracking error was measured by calculating the standard error of the regression.

Source: Vanguard calculations based on data from Morningstar, Inc., and the Kenneth R. French data library.

Cost

The importance of low-cost investment management has been identified by previous research. On average, we have found cost to have a negative one-to-one relationship with excess returns: For every one-basis-point increase in the expense ratio, subsequent net excess returns tend to decrease by one basis point, on average.[14] To the extent an investor is subject to taxes, these are an additional form of cost that can further affect the odds of successfully using active management. Active funds tend to have larger tax impacts than market-cap-weighted index funds, on average.[15]

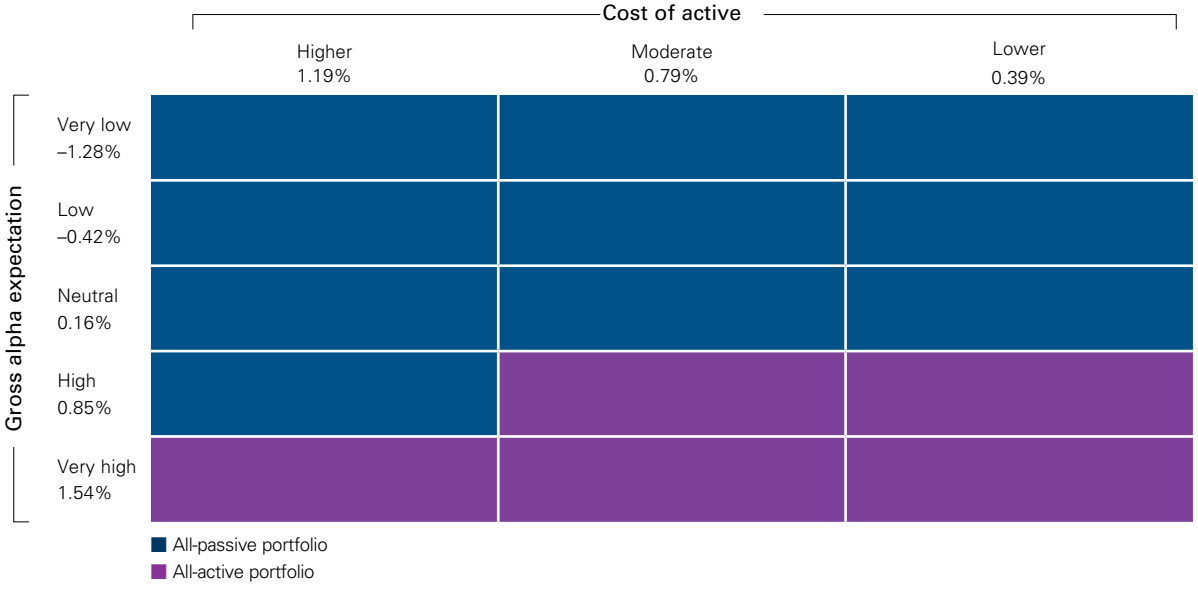
Gross alpha expectation addresses gross returns; however, in the end it is net returns (gross returns less cost) that matter to investors. In this analysis, we applied three separate tranches of cost—higher, moderate, and lower—to the entire population of simulated active funds:

- **Higher cost:** The median asset-weighted expense ratio + 0.40% (or 1.19%).
- **Moderate cost:** The median asset-weighted expense ratio of 0.79%.
- **Lower cost:** The median asset weighted expense ratio – 0.40% (or 0.39%).

Part one results: When alpha and cost are everything

When we run the two critical attributes of gross alpha expectation and cost through our model (with no consideration for active risk or active risk tolerance), we arrive at the results shown in **Figure 8**. It displays the outcomes of the 15 scenarios ranging from the lowest gross alpha expectation and higher-cost active managers (the top left corner of the grid) to the highest gross alpha expectation and lower-cost active managers (the bottom right corner) and calculates the projected allocation ranges resulting from each set of circumstances.

Figure 8. Potential active-passive allocations based on two characteristics



Note: This hypothetical illustration does not represent any particular investment.
Source: Vanguard.

14 See Wallick, Wimmer, and Balsamo (2015a) and Wallick, Wimmer, and Balsamo (2015b) for two examples.

15 Although tax costs vary among investors, countries, funds, and timeframes, the tax costs of active funds have been—in many periods—higher than those of index funds. See Donaldson et al. (2015) for further discussion. Therefore, an investor considering the use of low-cost active funds in a taxable account would likely need to assume a moderate or high level of total cost after taxes, and a taxable investor using moderate-cost funds would likely need to assume a very high level. Investors subject to tax would be well-served by following general principles of taxable asset location and using active funds in tax-deferred or tax-free (e.g., Roth IRA) accounts.

Two items are most striking. First, without a positive net alpha expectation an investor would be better off investing 100% in index funds. Second, without incorporating the impact of active risk and active risk tolerance, we are left with a binary choice of either 100% passive funds (when net alpha expectation is negative) or 100% active funds (when net alpha expectation is positive).

These simplified conditions result in no recommended active-passive mix but simply provide a measure of whether or not positive net alpha is expected. For example, the box at the intersection of high gross alpha expectation (0.85%) and moderate costs (0.79%) has a net alpha expectation of just 0.06% (0.85%–0.79%), yet the recommendation is all active funds.

An investor concerned about active risk might not allocate 100% to active funds in this instance. He or she might well incorporate some portion of index funds in order to moderate the risk. Part two of our analysis reflects this reality.

Part two analysis: Net alpha expectations plus risk considerations

Next, we added risk considerations (active risk level and tolerance for active risk) to net alpha expectations (gross alpha minus cost) for our active portfolio to reassess how the combination influences the active-passive decision.

We divided both active risk and active risk tolerance into 3 levels by expanding each of the previously determined 15 pairings (5 different gross alpha expectation levels x 3 different active risk tolerances) into an additional 9 subdivisions (3 different levels of active risk x 3 different levels of active risk aversion) for a total of 135 possible situations.

Active risk

Taking on active risk is a necessary condition for producing outperformance but obviously not a guarantee. We assessed a range of active risk (i.e., tracking error) for the funds, established three active-risk cohorts, and showed how these levels calibrated to recent history:[16]

- **Higher active risk:** The median of the one-third of funds with the highest active risk (or 5.12%).
- **Moderate active risk:** The median of the one-third of funds with moderate active risk (or 3.45%).
- **Lower active risk:** The median of the one-third of funds with the lowest active risk (or 2.41%).

Active risk tolerance

The critical final element is active risk tolerance, essentially an investor's ability to handle a given level of alpha variability through time and willingness to accept the uncertainty of achieving outperformance. We used a risk aversion parameter within a utility function to penalize alpha variability by differing amounts:

- **Higher active risk tolerance:** a lesser risk aversion penalty in the utility function.
- **Moderate active risk tolerance:** a moderate risk aversion penalty in the utility function.
- **Lower active risk tolerance:** a greater risk aversion penalty in the utility function.

The use of a utility function with an embedded risk tolerance parameter may be an abstract concept to some. Here, it enables us to quantify different investor risk preferences for dealing with the uncertainty of active management, where risk is associated with the alpha variability of the active portfolio.[17]

This portion of the analysis allows us to calculate utility-adjusted wealth (rather than simply total wealth, as we saw in part one) and understand the risk-driven trade-offs between active management (which has some degree of manager uncertainty) and passive management (which has little manager uncertainty) for a range of active risk tolerance levels. Without it, the model would produce only all-or-nothing active or passive allocations.

16 Active risk was calculated using the standard error of the residuals of the Fama-French three-factor regressions.

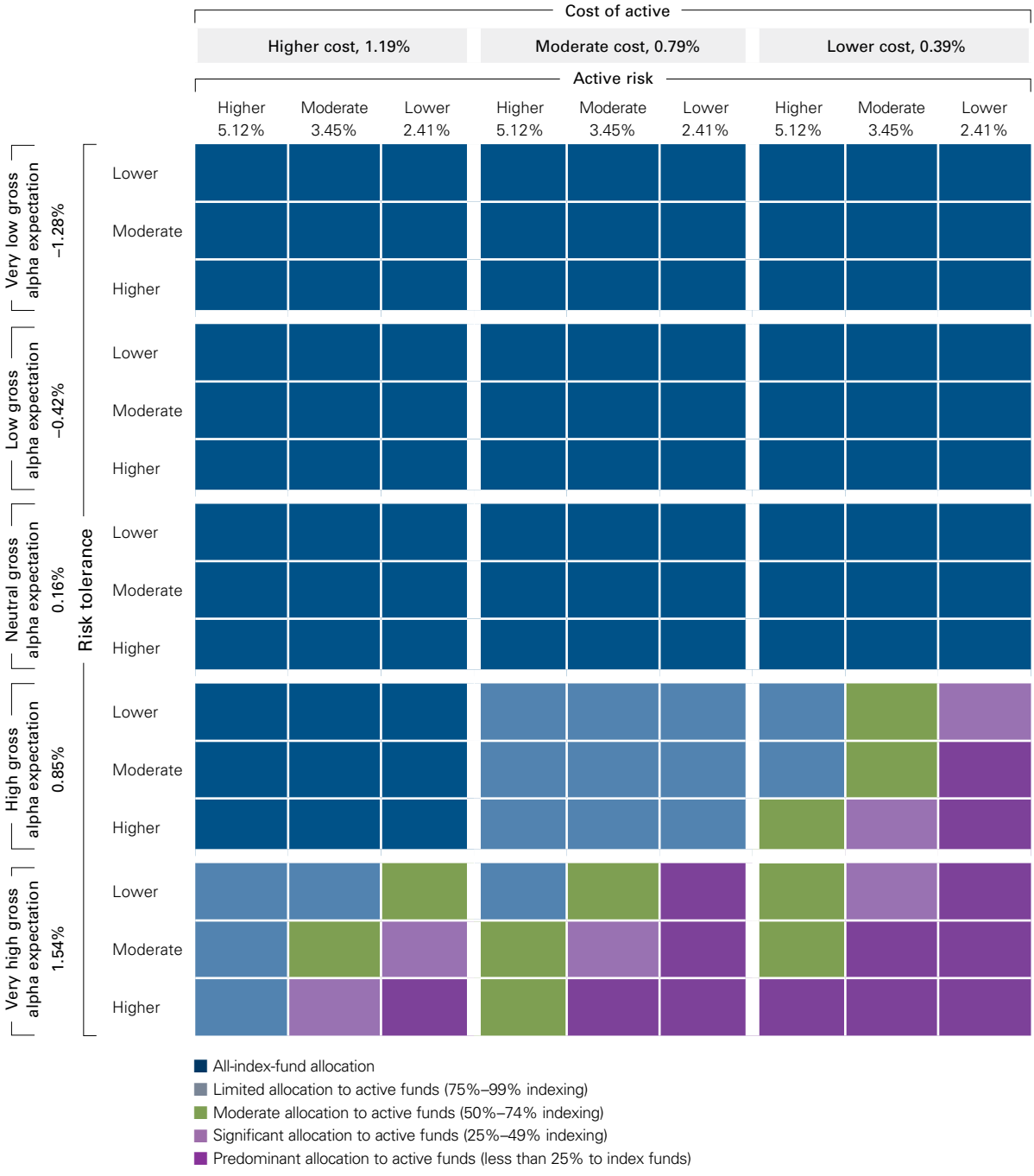
17 The actual parameters used in the model for this example were 14, 10, and 6 for lower, moderate, and higher active risk tolerance. Investors have a lower tolerance for active risk than beta risk, hence the relatively high penalties. An investor with a lower level of active risk tolerance could be thought of as one who has either a strictly defined tracking error budget or a lower likelihood of remaining invested in an active fund during periods of underperformance. An investor with a higher level, on the other hand, would likely have a more flexible active risk budget (if one is used at all) and the expectation to remain invested during periods of underperformance.

Part two results: When net alpha and risk considerations lead to a wider range of outcomes

Combining the elements of active risk and active risk tolerance with gross alpha expectation and cost results in four variables, each with three different measurement levels. This 5x3x3x3 structure is illustrated in **Figure 9**.

All the scenarios for investors with neutral (or worse) gross alpha expectations remain unchanged from the part one analysis. If outperformance is the goal, cost, active risk level, and active risk tolerance do not supersede the importance of identifying talent.

Figure 9. Potential active-passive allocations based on four characteristics



Note: This hypothetical illustration does not represent any particular investment.

Source: Vanguard.

For those investors with high and very high gross alpha expectations (the expectation that they will select among the top two-thirds or one-third of all managers), indexing still makes up a sizable portion of many allocations. Cost remains an influencing factor, as does aversion to active risk. But when higher assumptions for gross alpha are combined with lower assumptions for cost, active risk, and active risk aversion, active allocations predominate.

This approach allows us to move from simple binary solutions of all-active or all-passive investment to a more nuanced set of results that demonstrates the trade-off between net alpha expectations and tolerance for active risk. Furthermore, the quantitative nature of our framework discloses otherwise embedded assumptions and enables investors to assess a range of inputs.

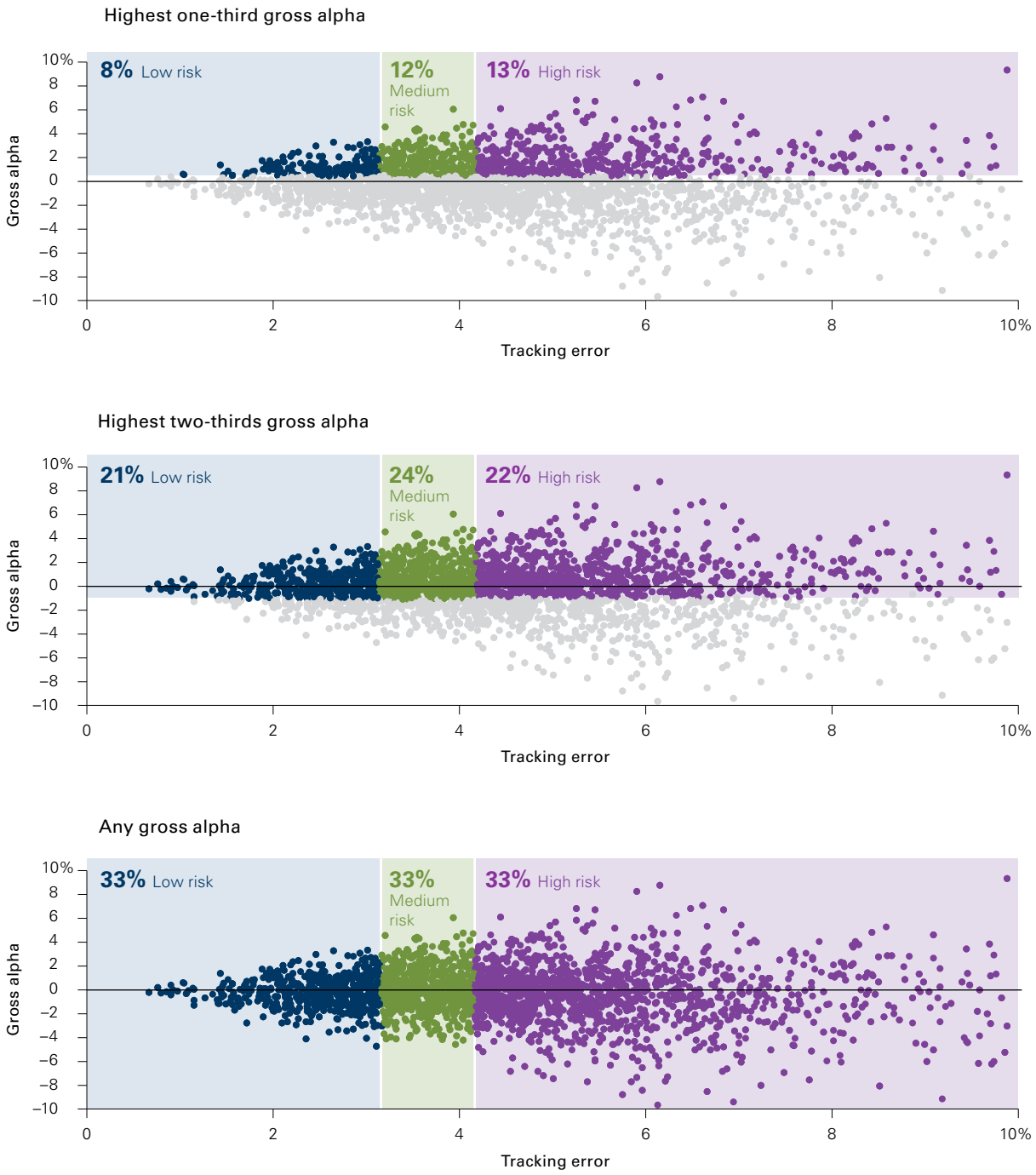
Conclusion

Our simulation analysis identifies three overall conclusions. First, indexing can be a valuable starting point for all investors. Per our research, a lack of conviction in identifying active manager talent results in an all-indexing solution.

Second, it reiterates prior Vanguard research demonstrating that the use of active management is dependent on talent, cost, and patience (represented in our analysis by gross alpha expectation, cost, and active risk tolerance). The greater an investor's ability to identify talented active managers, access them at a low cost, and remain patient amid the inconsistency of alpha through time, the greater the suggested allocation to active funds.

Third, investors considering both active and passive investments will benefit from explicitly identifying assumptions regarding four key components: gross alpha, cost, manager risk, and risk tolerance. Because this tailored approach is based on an investor's specific expectations, there will be no one-size-fits-all result.

Appendix A: The percentage of U.S. equity funds meeting gross alpha and active risk criteria over the ten years ended June 30, 2016



Notes: Data are for the ten-year period from July 1, 2006, through June 30, 2016, and represent active equity funds with at least 36 months of history available to U.S. investors in the following categories: small-cap value, small-cap growth, small-cap blend, mid-cap value, mid-cap growth, mid-cap blend, large-cap value, large-cap growth, and large-cap blend. Funds that died or merged were included in the analysis. The oldest and lowest-cost single share class was used to represent a fund when multiple share classes existed. Asset-weighted results were calculated using each fund's average reported monthly assets. Alpha was calculated by regressing monthly gross returns against the Fama-French three factors of small minus big, high minus low, and excess return on the market over the risk-free rate. Tracking error was measured by calculating the standard error of the regression.

Source: Vanguard calculations based on data from Morningstar, Inc., and the Kenneth R. French data library.

Appendix B: Factor allocations in the active-passive framework

This paper focuses on alpha expectations, but our framework can also accommodate the active returns and risk that result from factor exposures. In that case, rather than set expectations for gross alpha levels, their associated active risk levels, and cost, the investor estimates those variables for factors.[18] The due diligence process for factors has many similarities with the search for alpha. For example, the investor must:

- Assess the talent of the people designing the strategy and understand how it's implemented.
- Have a logical rationale for why the active investment process will have a reasonable chance of producing a certain outcome.
- Evaluate to what extent management and various implementation costs will erode the strategy's desired benefit.
- Have the patience to handle sharp and prolonged periods of underperformance relative to a broad, cap-weighted index.[19]

Our framework allows investors to consider alpha and factor-seeking strategies together to determine the mix consistent with their goals, beliefs, and circumstances. A critical step is to assess the combined attributes of the active allocation when it consists of multiple alpha or factor strategies. This analysis will reveal whether the strategies' aggregate active exposures (their combined security, sector, factor, country, and regional weights, for example), reflect the investor's objectives. If they do not, it can help determine the trade-offs inherent in shifting to more suitable weightings.

Finally, as part of ongoing due diligence, the investor must regularly gauge whether the active results have been and will likely continue to be driven by the desired exposures in the most cost-effective way.[20]

Appendix C: Quantitative approach to the active-passive decision

Mathematically, the active-passive allocation decision is the solution to the following optimization problem:

$$w_A^* = argmax\left[E\left(U\left(\prod_t^T (1 + r_t^A \cdot w_A + r_t^P \cdot (1 - w_A))\right)\right)\right]$$

subject to $0 \le w_A \le 1$

where w_A is the allocation to active; $r_t^A \sim$ N(alpha,tracking error^2) is the stochastic return process for the active manager; r_t^P is the return of the passive benchmark; and $U(\cdot)$ is the utility function. The *argmax* $(\cdot)$ function solves for the optimal allocation w_A^* corresponding to the maximum of the average (across 10,000 simulations) utility/risk-adjusted cumulative portfolio return over the investment horizon T.

The utility function specification is of the Constant Relative Risk Aversion type, $U(x) = \frac{(x)\,1-y}{1-y}$, where y is the risk-aversion parameter that measures the level of active risk tolerance an investor is willing to accept in the portfolio.

18 For general information on the topic of factor-based investing, see Pappas and Dickson (2015). For a detailed discussion of important considerations for equity factor-based investment vehicles, see Grim et al. (2017).

19 This would include any strategy labeled as a strategic or smart beta index, because by design, the indexes take on active risks by intentionally choosing weights that differ from a broad, cap-weighted index. For more information, see Philips, Bennyhoff, Kinniry, Schlanger, and Chin (2015).

20 For an example of a famous ex post assessment conducted on the Norwegian Government Pension Fund, see Ang, Goetzmann, and Schaeffer (2009).

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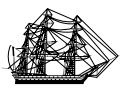

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